

## **Mastery Professional Development**

### *Number, Addition and Subtraction*



#### 1.27 Negative numbers: counting, comparing and calculating

Teacher guide | Year 5

##### **Teaching point 1:**

Positive and negative numbers can be used to represent change.

##### **Teaching point 2:**

Our number system includes numbers that are less than zero; these are negative numbers. Numbers greater than zero are positive numbers.

##### **Teaching point 3:**

The negative/minus symbol (–) is placed before a numeral to indicate that the value is a negative number.

##### **Teaching point 4:**

Negative numbers can be shown on horizontal scales; numbers to the left of zero are negative (less than zero) and numbers to the right of zero are positive (greater than zero). The larger the value of the numeral after the negative/minus symbol, the further the number is from zero.

##### **Teaching point 5:**

Knowledge of the positions of positive and negative numbers in the number system can be used to calculate intervals across zero.

##### **Teaching point 6:**

Negative numbers are used in coordinate and graphing contexts.

## Overview of learning

In this segment children will:

- learn that our number system includes numbers that can be positive or negative
- develop an understanding of negative numbers in a range of contexts, including scales and changing quantities
- learn that the symbol used before a numeral to indicate that it is negative can mean 'subtract' and 'negative'
- use a number line that extends into negative numbers
- compare and order negative and positive numbers
- learn to calculate intervals across zero
- apply their understanding of positive and negative numbers to graphing contexts.

This segment introduces children to negative numbers for the first time. The first teaching point uses a hands-on approach to give children their first 'taste' of negative numbers, using number sentences to represent changes in quantities; for example, we begin with some marbles in a bag, then put in three more marbles (3), and then take out four marbles ( $-4$ ); we can represent the change in the number of marbles as  $3 - 4 = -1$ .

*Teaching points 2, 3, and 4* explore negative numbers in the context of ordinality and scales. Here, children will build on their understanding of how numbers are ordered and positioned relative to one another, introducing negative numbers as those less than zero (and positive numbers as those greater than zero). Children will explore these relationships through a range of contexts, including temperature and elevation compared to sea level, before learning to calculate intervals across zero in *Teaching point 5*, using a number line as a scaffold.

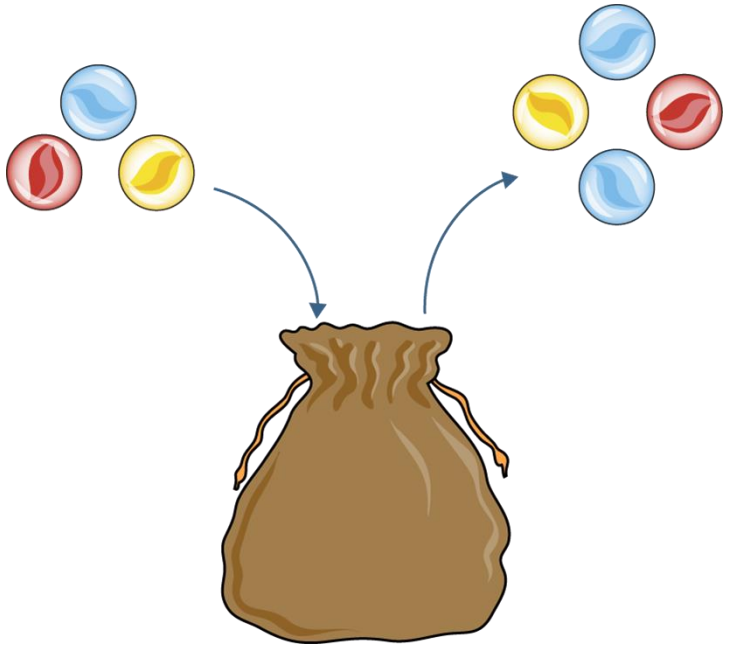
A common mistake is for children to think that fractions and decimals are negative numbers. Using a number line to place and order numbers will address this misconception.

The final teaching point focuses on application of children's understanding of negative numbers to graphing contexts, including working with all four quadrants of the coordinate grid.

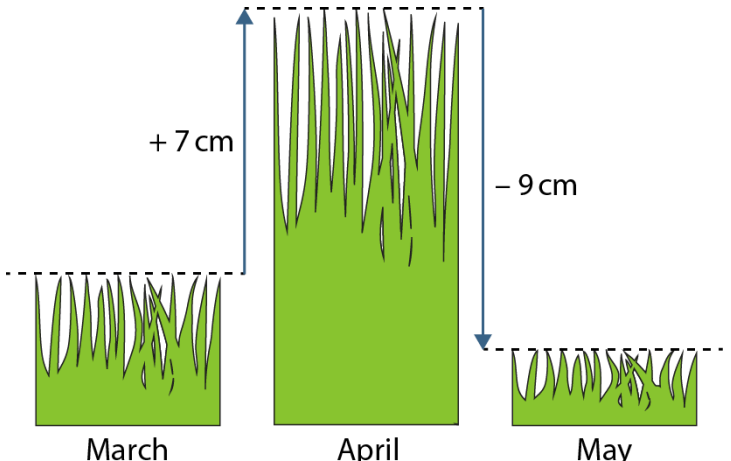
**Teaching point 1:**

Positive and negative numbers can be used to represent change.

**Steps in learning**

|            | Guidance  | Representations  |
|------------|---|--|
| <b>1:1</b> | <p>Before looking at negative numbers in the context of scales (for example, temperature) and ordinality, it is worth taking a more hands-on approach to introduce children to the idea of a negative number.</p> <p>Begin by playing a game. Start with an opaque bag of marbles (or other items) plus some spare marbles; do not tell the class how many marbles are in the bag, just say there are 'some marbles'.</p> <ul style="list-style-type: none"> <li>• Ask a child (A) to say 'Go!'</li> <li>• Ask another child (B) to tell you a number of marbles to <i>add</i> to the bag (let's assume that child B says 'Three').</li> <li>• Add that number of marbles to the bag and ask 'Are there more marbles in the bag now than when child A said "Go!" or fewer?' Children should confidently identify that there are more.</li> <li>• Ask 'How many more?' Children should confidently identify the number (here, three). Write this number on the board (3).</li> <li>• Now ask a third child (C) to tell you a number of marbles to <i>take out</i> of the bag (let's assume that child C says 'four').</li> <li>• Again, ask 'Are there more marbles in the bag now than when Child A said "Go!" or fewer?' With the exemplar numbers here (three and four), children should realise that you've taken out more marbles than you put in, so there should be fewer in the bag than when child A said 'Go!'.</li> <li>• Ask 'How many fewer?' Children should be able to identify how many</li> </ul> |  <p><math>3 - 4 = -1</math></p> |


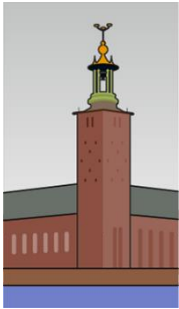
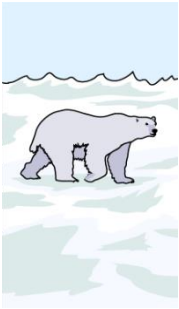
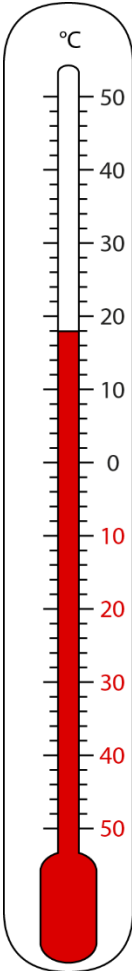
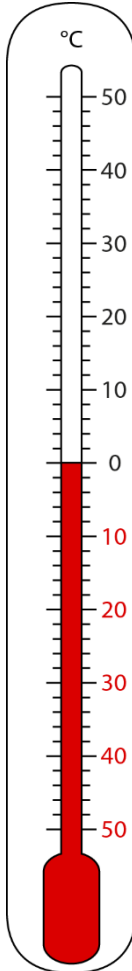
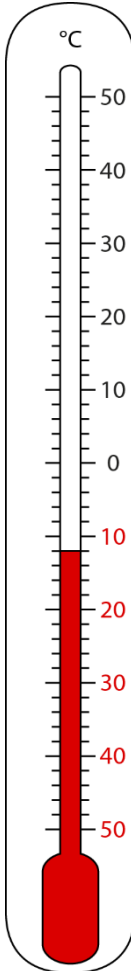

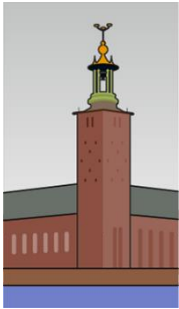
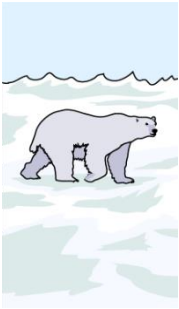
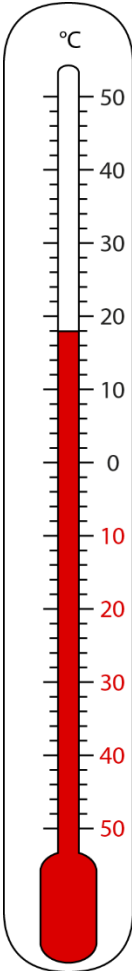
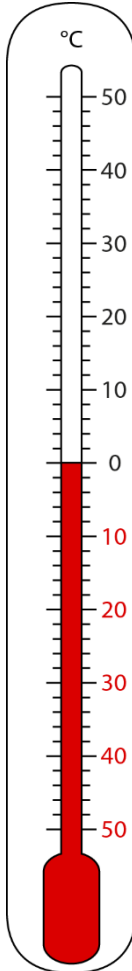
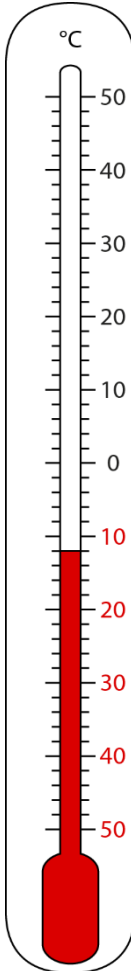

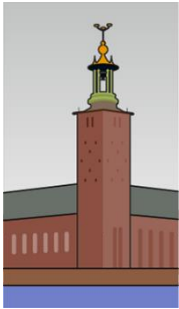
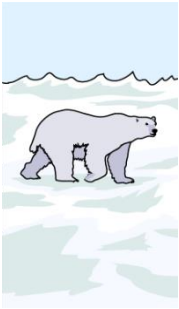
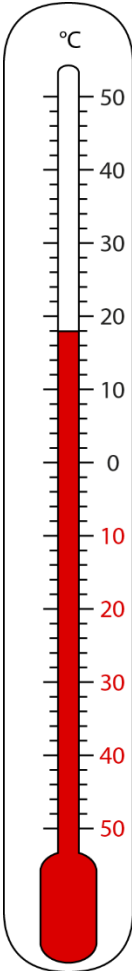
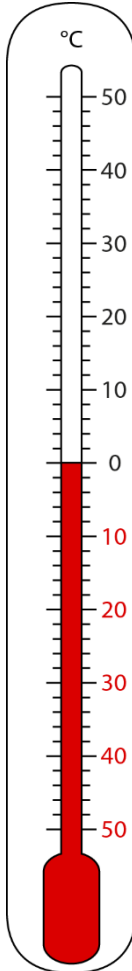
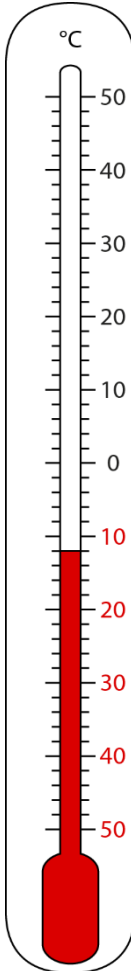
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|            | <p>fewer (here, one). Add to the text on the board; in this case: <math>3 - 4 = -1</math></p> <p>*If initially a child does not suggest removing a greater number than was added, simply play the game again until this arises naturally.</p>  |  |
| <b>1:2</b> | <p>Repeat the game in step 1:1 several times, until children are familiar with expressions of the form <math>3 - 4 = -1</math>. Note that the numbers recorded during the game don't represent the <i>actual number of marbles</i> (we don't know how many were in the bag at the start of each game) but instead represent <i>change</i>. The symbols represent the actions on the marbles. As you continue playing the game (allowing inclusion of examples when fewer marbles are taken out than put in, e.g. <math>3 - 1 = 2</math>), children will begin to see the pattern and understand the significance of the '-' symbol before, for example, the '1' in <math>3 - 4 = -1</math></p> <p>To draw this out, after each game, ask children to describe what each part of the equation represents, for example, for <math>3 - 4 = -1</math>:</p> <ul style="list-style-type: none"> <li>• 'The "3" represents the number of marbles added to the bag.' (It does not represent the number of marbles in the bag).</li> <li>• 'The "-4" represents the number of marbles taken out of the bag.'</li> <li>• 'The "-1" represents the change in the number of marbles.'</li> </ul> |  |
| <b>1:3</b> | <p>Once children begin to see the pattern, you could ask them to:</p> <ul style="list-style-type: none"> <li>• write number sentences for situations where they have to <i>imagine</i> a game that you describe, for example: <ul style="list-style-type: none"> <li>• 'Jo said "Go!" I put six marbles in the bag, then I took eight marbles out of the bag.'</li> </ul> </li> </ul>  |  |

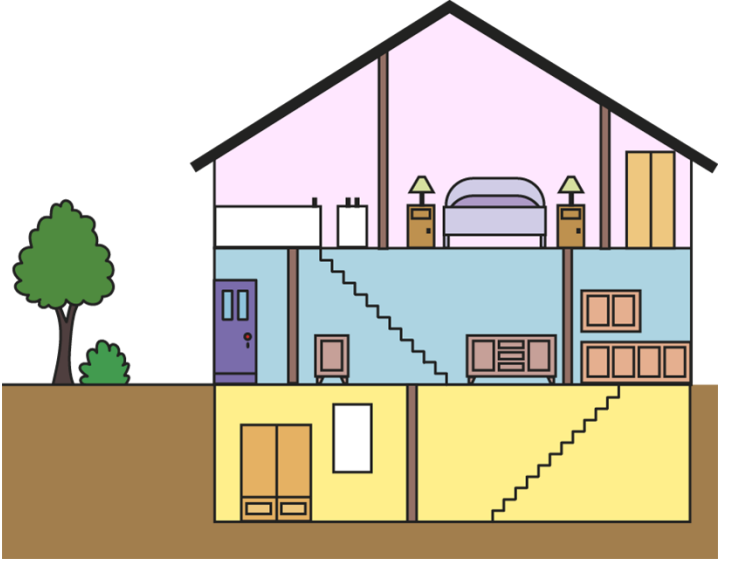
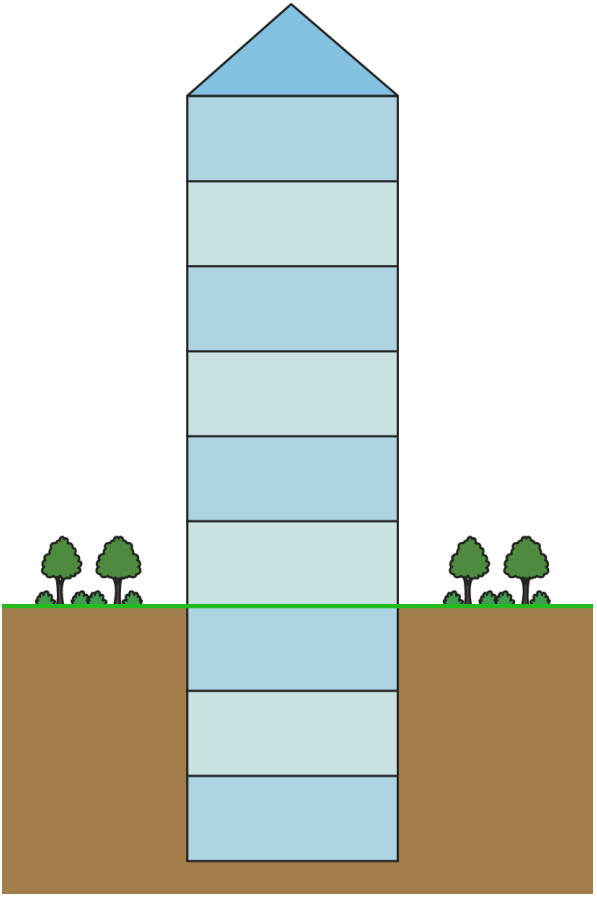
|            |   |   |
|------------|---|---|
|            | <ul style="list-style-type: none"> <li>describe a game for which you have provided a number sentence</li> <li>complete gaps in number sentences linked to the game, for example:<br/> <math>5 - \square = 3</math><br/> <math>5 - \square = -3</math> </li> </ul>   |   |
| <b>1:4</b> | <p>Provide independent practice, also including some different contexts, for example: <i>There were some people on a bus. The bus stopped and eight people got on. At the next stop, three people got off. How many people are on the bus now compared to the start? Write an equation to show this.</i></p> <p>Note that throughout this teaching point, we always increase the amount first and then decrease the amount, for example, adding marbles to the bag then taking marbles out (<math>3 - 4 = -1</math>), rather than taking marbles out then adding some in (<math>-4 + 3 = -1</math>). Mathematically there is no need to always increase/add first, then decrease/subtract, but the examples given in this teaching point meet the aim of introducing the idea of negative numbers in the context of change, so we suggest using only the 'increase then decrease' structure for simplicity.</p> | <p><i>'Between March and April, the grass in Bill's garden grew 7 cm. At the beginning of May, Bill cut the grass by 9 cm. How long is the grass in May compared to March?'</i></p>  <p>March                      April                      May</p> <p><math>7 - 9 = -2</math></p> |

**Teaching point 2:**

Our number system includes numbers that are less than zero; these are negative numbers. Numbers greater than zero are positive numbers.

**Steps in learning**

|  | Guidance  | Representations   |                 |                   |            |   |   |   |  |  |  |
|--|---|---|-----------------|-------------------|------------|---|---|---|--|--|--|
| <b>2:1</b>   | <p>Begin this teaching point by focusing on the relationship between various numbers and zero. Using a vertical scale, such as a thermometer, explore ideas of zero, values above zero and values below zero. At this stage, do not use the negative/minus symbol to indicate numbers below zero (although children have had some exposure to the symbol in <i>Teaching point 1</i>, it will be introduced in the context of scales and ordinality in <i>Teaching point 3</i>).</p> <p>Throughout this and the next teaching point, restrict examples to vertical scales; here, children will use the language 'above' and 'below' zero when identifying positive and negative numbers. Horizontal scales will be included from <i>Teaching point 4</i> onwards.</p> <p>Show a range of temperatures (as in the examples opposite) and ask children whether each is above or below (or indeed equal to) <math>0^{\circ}\text{C}</math>. Ask them how they can tell whether a temperature is hotter or colder than <math>0^{\circ}\text{C}</math>, working towards the following understanding:</p> <ul style="list-style-type: none"> <li>For temperatures above zero, the further the temperature is from zero the hotter it is.</li> <li>For temperatures below zero, the further the temperature is from zero the colder it is.</li> </ul> <p>Set your discussion in the context of the SI unit for temperature (Celsius), linking <math>0^{\circ}\text{C}</math> to the freezing point of water and how temperatures may be different in different locations.</p> | <p>Temperatures in different places on a given day:</p> <table> <tr> <th>London, England</th><th>Stockholm, Sweden</th><th>North Pole</th></tr> <tr> <td></td><td></td><td></td></tr> <tr> <td></td><td></td><td></td></tr> </table> | London, England | Stockholm, Sweden | North Pole |  |  |  |  |  |  |
| London, England  | Stockholm, Sweden   | North Pole  |                 |                   |            |   |   |   |  |  |  |
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|-----|---|---|
|     | Throughout this segment, ensure that children use the correct language <i>'degrees Celsius'</i> rather than, for example, <i>'degrees C'</i> .  |   |
| 2:2 | <p>Now explore a different context: buildings with floors above and below ground. Explore where the ground floor is and what happens if you go up/down the stairs or up/down in a lift.</p> <p>Ask children which floor they think could be labelled '0', and discuss how usually, in the UK, the 'ground' floor is labelled as zero, and that we count up from zero using each subsequent number to label the floors above the ground floor. Ask <i>'Which is floor three?'</i>, <i>'Which is the fifth floor?'</i> etc.</p> <p>Then ask about the floors below zero/ground level. Discuss what these could be called (for example, <i>'first basement, second basement...'</i> or <i>'first below ground, second below ground...'</i>).</p> |   |

**2:3**

Discuss a further context for children to explore values above and below zero, such as elevation above and below sea level. Draw attention to where zero might be placed and what zero might mean. For now, continue to avoid use of the negative/minus symbol.

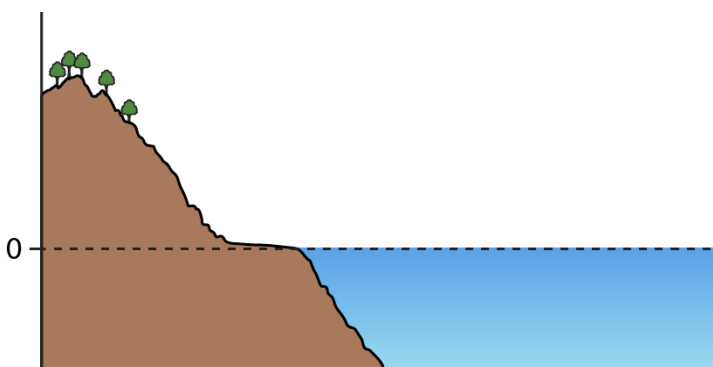
Now that several examples have been explored, explain that we use a set of numbers to label below zero, and that these are called negative numbers and are *less than* zero. Also introduce the term 'positive numbers', explaining that these are *greater than* zero.

Use the generalised statements:

- **'Negative numbers are below zero.'**
- **'Negative numbers are less than zero.'**
- **'Positive numbers are above zero.'**
- **'Positive numbers are greater than zero.'**

Emphasise that zero is neither positive nor negative.

Note that the terms 'below' and 'above' are prepositions linking only to vertical scales, but these are useful terms for children at this stage, especially since we began the discussion with contexts such as temperature and elevation. Once horizontal scales are included in the discussion (*Teaching point 4*) 'below' and 'above' must be changed to the new prepositions 'to the left of' and 'to the right of', and there we suggest moving to the more mathematically rigorous 'less than' and 'greater than'. However, you can begin to model and include the 'less than' and 'greater than' language here too.

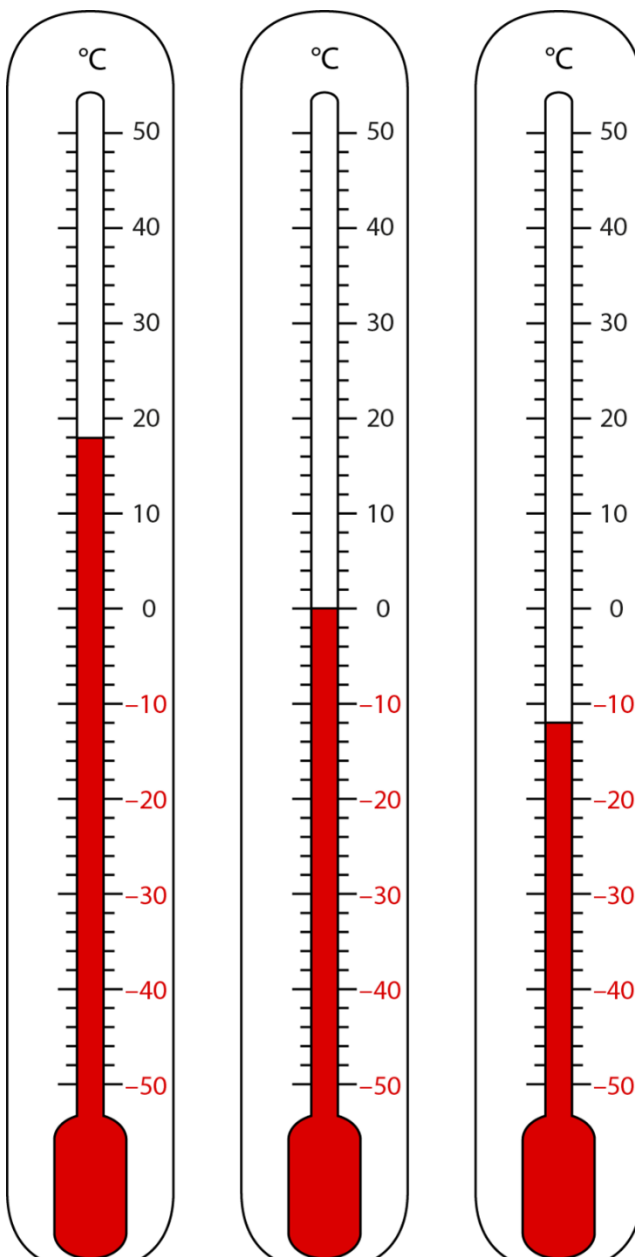


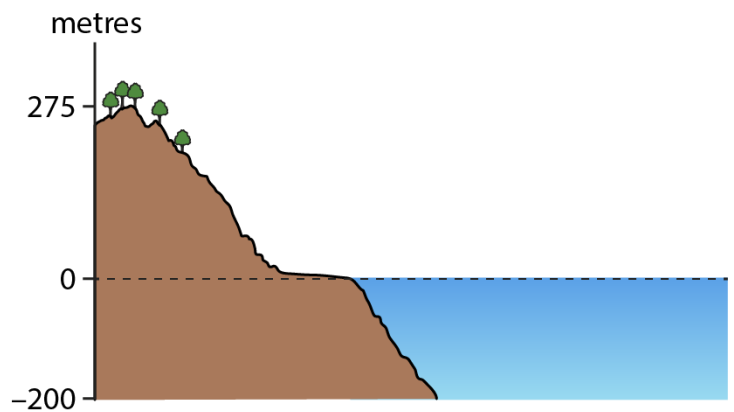
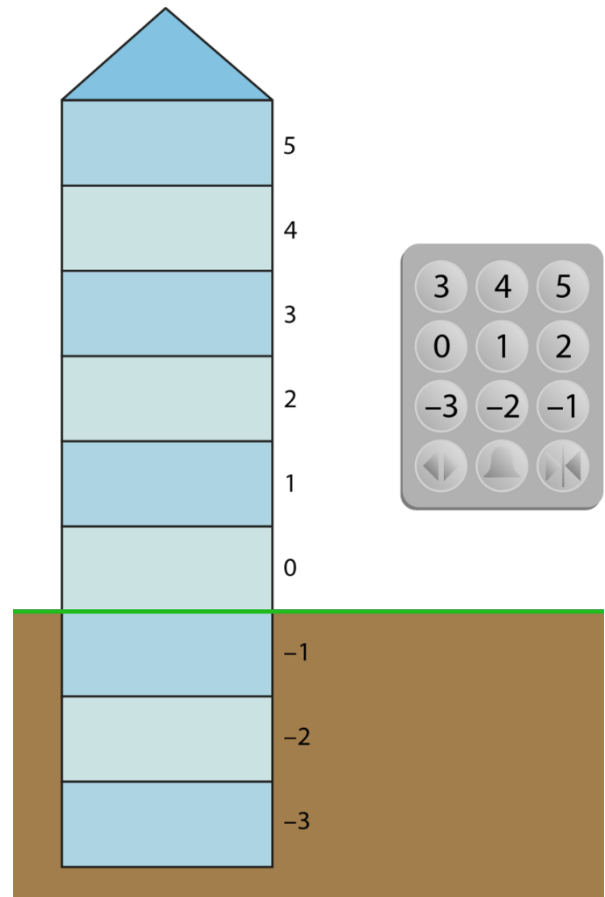


**Teaching point 3:**

The negative/minus symbol (–) is placed before a numeral to indicate that the value is a negative number.

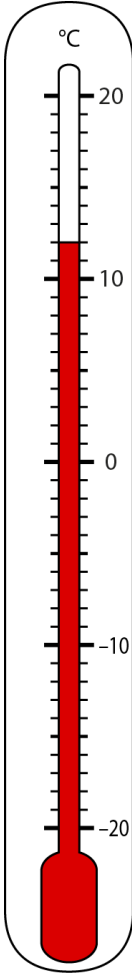
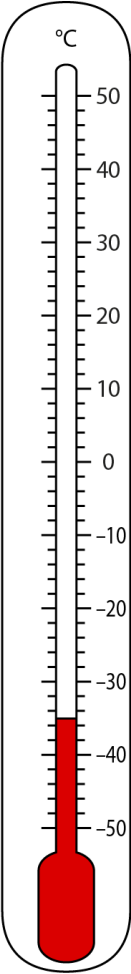
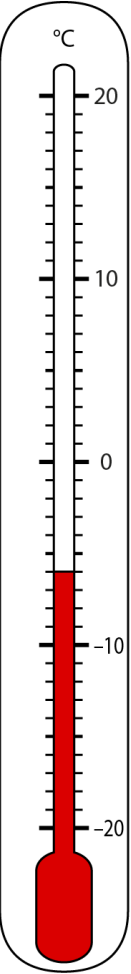
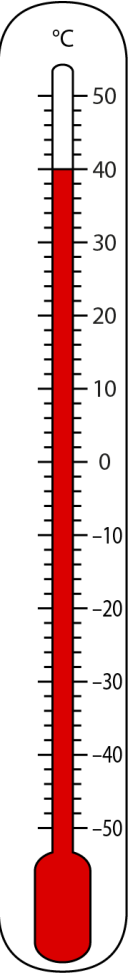
**Steps in learning**

|            | Guidance  | Representations   |
|------------|---|---|
| <b>3:1</b> | <p>Now revisit the same contexts that you used in <i>Teaching point 2</i>, but this time introduce the negative/minus symbol to indicate numbers less than zero. Explain that the symbol we use to denote a negative number is the same as the one used to represent subtraction (this links to the work done in <i>Teaching point 1</i>). Similarly, positive numbers could be, but aren't usually, shown with a positive/plus symbol before the numeral.</p> <p>Practise reading values above and below zero using the correct language. Explain, for example, that:</p> <ul style="list-style-type: none"> <li>• '–1' can be read as '<i>negative one</i>'; the '–' symbol indicates that the number is negative</li> <li>• '5' is usually just read as '<i>five</i>' (rather than '<i>positive five</i>'); there is no '–' symbol, so the number is positive.</li> </ul> <p>Identify the negative numbers on the thermometers and other scales, continuing to use the generalised statements from step 2:3:</p> <ul style="list-style-type: none"> <li>• '<b>Negative numbers are below zero.</b>'</li> <li>• '<b>Negative numbers are less than zero.</b>'</li> <li>• '<b>Positive numbers are above zero.</b>'</li> <li>• '<b>Positive numbers are greater than zero.</b>'</li> </ul> |  |

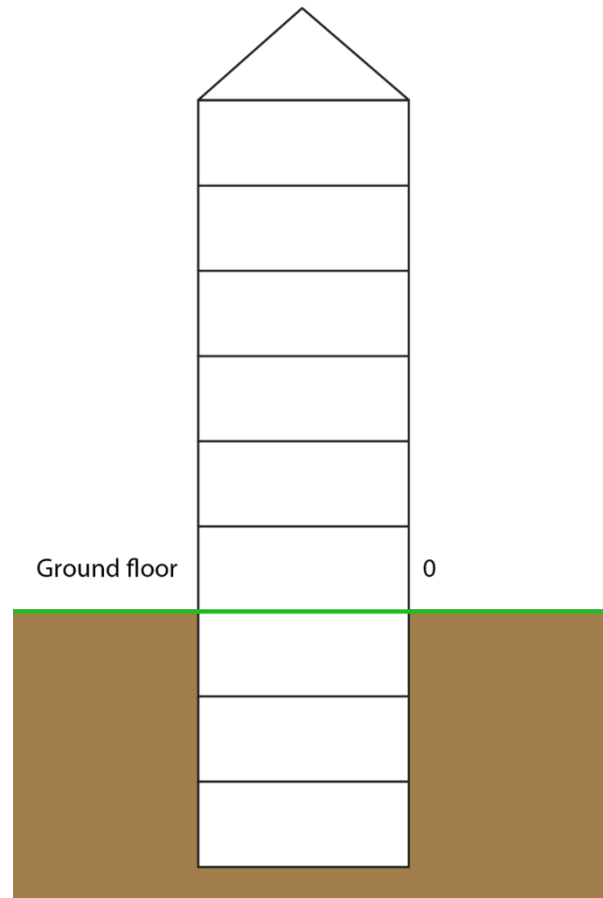


**3:2** For each context, encourage children to use full sentences to generalise about the scales:

- 'Temperatures warmer than zero degrees are positive.'
- 'Temperatures colder than zero degrees are negative.'
- 'Zero degrees is neither positive nor negative.'

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|     | <p>Similarly:</p> <ul style="list-style-type: none"> <li>• 'Floors above the ground floor are positive.'</li> <li>• 'Floors below the ground floor are negative.'</li> <li>• 'The ground floor is labelled zero; it is neither positive nor negative.'</li> </ul> <p>And:</p> <ul style="list-style-type: none"> <li>• 'Places above sea level have a positive elevation.'</li> <li>• 'Places below sea level have a negative elevation.'</li> <li>• 'Sea level is labelled zero; it is neither positive nor negative.'</li> </ul> |  |
| 3:3 | <p>Provide children with practice reading scales and writing negative numbers, as shown opposite.</p>  | <ul style="list-style-type: none"> <li>• 'Write down the temperature shown on each thermometer, in degrees Celsius.'</li> </ul> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <div style="border: 1px solid black; width: 60px; height: 25px; margin: 5px auto;"></div> </div> <div style="text-align: center;">  <div style="border: 1px solid black; width: 60px; height: 25px; margin: 5px auto;"></div> </div> <div style="text-align: center;">  <div style="border: 1px solid black; width: 60px; height: 25px; margin: 5px auto;"></div> </div> <div style="text-align: center;">  <div style="border: 1px solid black; width: 60px; height: 25px; margin: 5px auto;"></div> </div> </div> |

- 'Nicky gets into a lift at the ground floor. She presses the button labelled '-2'. Shade in the floor the lift takes her to.'



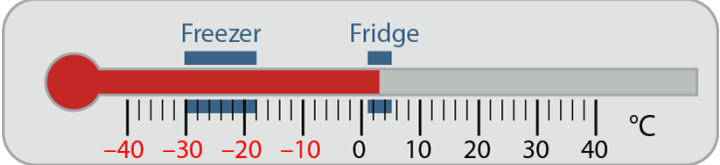
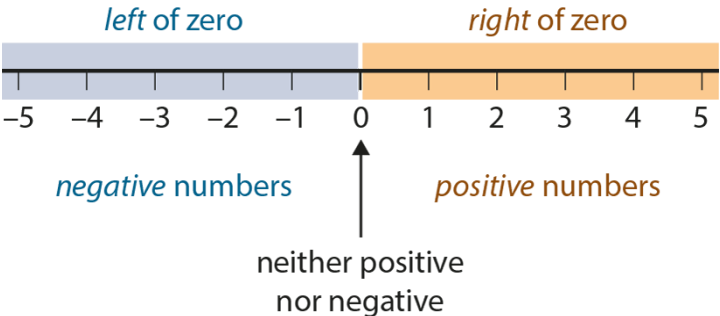
- 'Match each elevation, relative to sea level, to the correct object.'

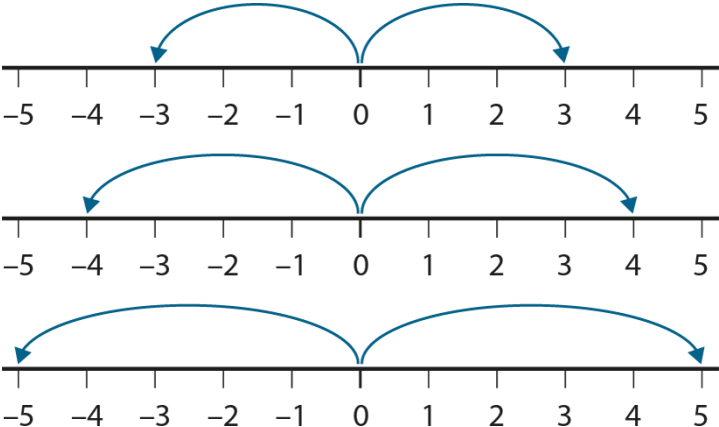
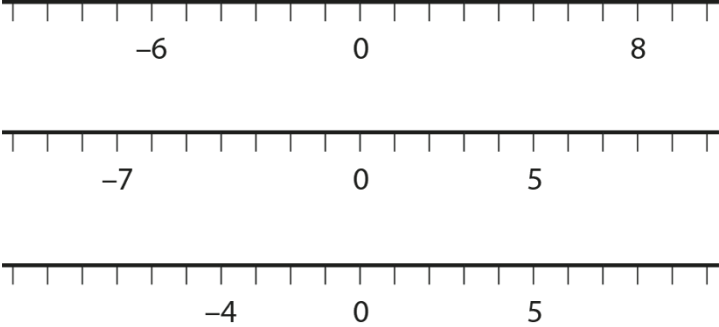
| Elevation | Object    |
|-----------|-----------|
| 15 m      | submarine |
| 0 m       | kite      |
| -30 m     | boat      |

**Teaching point 4:**

Negative numbers can be shown on horizontal scales; numbers to the left of zero are negative (less than zero) and numbers to the right of zero are positive (greater than zero). The larger the value of the numeral after the negative/minus symbol, the further the number is from zero.

**Steps in learning**

|            | Guidance  | Representations  |
|------------|---|--|
| <b>4:1</b> | <p>In this teaching point, children will learn to place numbers relative to one another and to zero on horizontal number lines.</p> <p>Begin by introducing horizontal scales in context, again identifying where the negative numbers are relative to zero. Then show and discuss a horizontal number line. Discuss how the generalised statements we used for vertical scales need to change, but the language of 'less than' and 'greater than' remains the same:</p> <ul style="list-style-type: none"> <li>• <b>'Negative numbers are below zero.'</b> becomes <b>'Negative numbers are to the left of zero.'</b></li> <li>• <b>'Positive numbers are above zero.'</b> becomes <b>'Positive numbers are to the right of zero.'</b></li> </ul> <p>This should then lead to the following generalisations:</p> <ul style="list-style-type: none"> <li>• <b>'Negative numbers are less than zero.'</b></li> <li>• <b>'Positive numbers are greater than zero.'</b></li> </ul> | <p>Fridge/freezer thermometer:</p>  <p>Horizontal number line:</p>  |
| <b>4:2</b> | <p>Now practise counting back in ones through zero, supported by the number line: '<i>...three, two, one, zero, negative one, negative two, negative three...</i>' Also practise counting forward in ones through zero, starting at a negative number: '<i>...negative three, negative two, negative one, zero, one, two, three...</i>'</p>   |  |

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|            | This will give children a sense of the relative positions of the numbers.  |   |
| <b>4:3</b> | <p>On the number line, show that a positive number and a negative number can be the same distance away from zero; for example, <math>-3</math> and <math>(+)3</math> are both the same distance from zero. Then work through a sequence of numbers showing that the greater the value of the digit (whether the number is positive or negative), the further the number is from zero. Work towards use of the following generalisation: <b>'For both positive and negative numbers, the larger the value of the number, the further it is from zero.'</b></p> <p>Provide children with practice:</p> <ul style="list-style-type: none"> <li>identifying which number in a given pair is closer to/further from zero; initially they can use the number line for support, but should ultimately be able to work without it</li> <li>completing expressions using less than and greater than symbols (<math>&lt;</math> and <math>&gt;</math>).</li> </ul> | <p>Distance from zero:</p>  <p>Example practice:</p> <ul style="list-style-type: none"> <li>'For each number line, circle the number that is further from zero.'</li> </ul>  |

- 'For each pair, put a tick in the correct column.'

| Number pair |      | Positive number further from zero | Negative number further from zero | Both numbers same distance from zero |
|-------------|------|-----------------------------------|-----------------------------------|--------------------------------------|
| -6          | 12   | ✓                                 |                                   |                                      |
| -12         | 6    |                                   |                                   |                                      |
| -6          | 6    |                                   |                                   |                                      |
| 10          | -1   |                                   |                                   |                                      |
| 10          | -10  |                                   |                                   |                                      |
| 10          | -100 |                                   |                                   |                                      |

- 'Fill in the missing symbols.' (<, > or =)

$1 \bigcirc 2$

$-1 \bigcirc -2$

$2 \bigcirc 3$

$-2 \bigcirc -3$

$3 \bigcirc 4$

$-3 \bigcirc -4$

$31 \bigcirc 29$

$-31 \bigcirc -29$

$31 \bigcirc 30$

$-31 \bigcirc -30$

$31 \bigcirc 31$

$-31 \bigcirc -31$

$31 \bigcirc 32$

$-31 \bigcirc -32$

$-1 \bigcirc 1$

$1 \bigcirc -1$

$-2 \bigcirc 1$

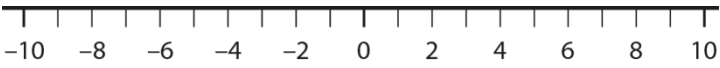
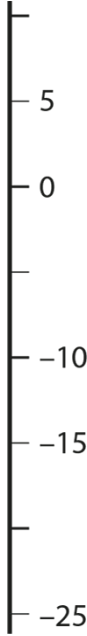

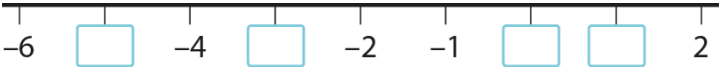
$2 \bigcirc -1$

$-3 \bigcirc 1$

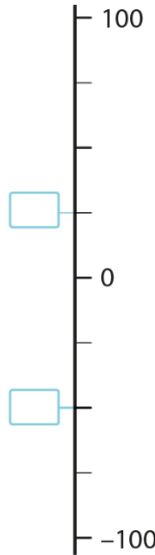

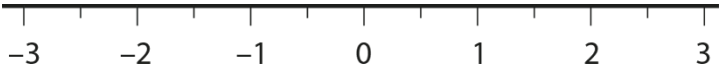
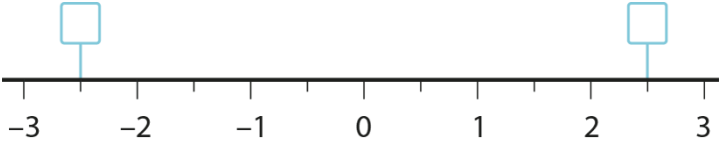
$3 \bigcirc -1$

$-4 \bigcirc 1$

$4 \bigcirc -1$

|                   |  |  |
|-------------------|--|--|
| <p><b>4:4</b></p> | <p>Now, using a variety of vertical and horizontal number lines, extend backward counting through zero to counting in multiples of different numbers, for example: '<i>...four, two, zero, negative two, negative four...</i>'. Similarly, extend forward counting from a negative number, for example: '<i>...negative four, negative two, zero, two, four...</i>'</p> <p>Vary the scales used as well as the amount of information given, as shown opposite.</p> | <p>Counting in multiples of two:</p>  <p>Counting in multiples of five:</p>  <p>Counting in multiples of eight:</p>  |
| <p><b>4:5</b></p> | <p>Now provide children with practice, including:</p> <ul style="list-style-type: none"> <li>identifying marked positions on number lines</li> <li>placing numbers on number lines.</li> </ul> <p>Use a variety of scales and orientations for the number lines, as shown opposite.</p>  | <p>• '<i>Fill in the missing numbers.</i>'</p>   |



|     |   |   |
|-----|---|---|
|     |   | <ul style="list-style-type: none"> <li>'Fill in the missing numbers.'</li> </ul>  <ul style="list-style-type: none"> <li>'Place negative two on this number line.'</li> </ul>    |
| 4:6 | <p>A common mistake is for children to think that common fractions, e.g. <math>\frac{2}{3}</math>, and decimal fractions, e.g. 0.5, are negative numbers.</p> <p>Highlight where fractions and decimal fractions are positioned on a number line and draw attention to where they are in relation to zero. Point out that they are positive, unless they are preceded by the negative/minus symbol.</p> | <ul style="list-style-type: none"> <li>'Place the following numbers on the number line.'</li> </ul> <p>0.5      <math>-\frac{1}{2}</math>      <math>\frac{1}{4}</math>      1.5      -2.5</p>  <ul style="list-style-type: none"> <li>'Fill in the missing numbers.'</li> </ul>  |

4:7

Now explore some of these concepts in context, as in the examples below/opposite.

- Discuss which place is colder: the South Pole or London, England. Then show the average monthly temperatures for the two places. Compare various numbers, positioning them on a vertical or horizontal number line and referring to the distance of each from zero. For example, explore the minimum and maximum values. Discuss why some people might think that  $-30^{\circ}\text{C}$  seems to be 'hotter' than  $-26^{\circ}\text{C}$  (South Pole, January) or why they might think that  $-36^{\circ}\text{C}$  is colder than  $-40^{\circ}\text{C}$  (South Pole, November). Work towards the following generalisations:

- ***'For negative temperatures, the further the number is from zero, the colder it is.'***
- ***'For positive temperatures, the further the number is from zero, the warmer it is.'***
- Explore the depth of a diver below sea level. Compare various depths, referring to the distance of each from zero. For example, ask 'Is the diver deeper when she's at negative fifty metres or when she's at negative thirty metres?' Work towards the following generalisations:
  - ***'When an object is below sea level, the further the number is from zero, the deeper the object.'***
  - ***'When an object is above sea level, the further the number is from zero, the higher the object.'***

Average monthly temperatures for the South Pole and London:

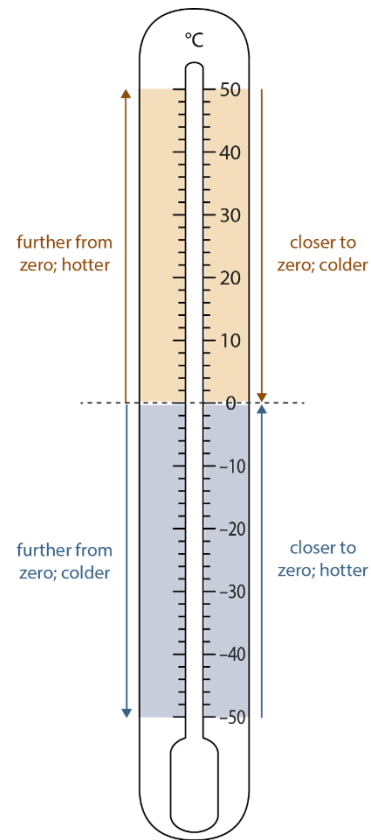
|     | South Pole                 |                            | London                     |                            |
|-----|----------------------------|----------------------------|----------------------------|----------------------------|
|     | Min ( $^{\circ}\text{C}$ ) | Max ( $^{\circ}\text{C}$ ) | Min ( $^{\circ}\text{C}$ ) | Max ( $^{\circ}\text{C}$ ) |
| Jan | -30                        | -26                        | 2                          | 8                          |
| Feb | -43                        | -38                        | 2                          | 8                          |
| Mar | -57                        | -50                        | 4                          | 11                         |
| Apr | -61                        | -53                        | 6                          | 14                         |
| May | -62                        | -54                        | 9                          | 18                         |
| Jun | -63                        | -54                        | 12                         | 21                         |
| Jul | -63                        | -55                        | 14                         | 23                         |
| Aug | -63                        | -55                        | 14                         | 23                         |
| Sep | -62                        | -54                        | 11                         | 20                         |
| Oct | -54                        | -48                        | 8                          | 16                         |
| Nov | -40                        | -36                        | 5                          | 11                         |
| Dec | -29                        | -26                        | 3                          | 8                          |

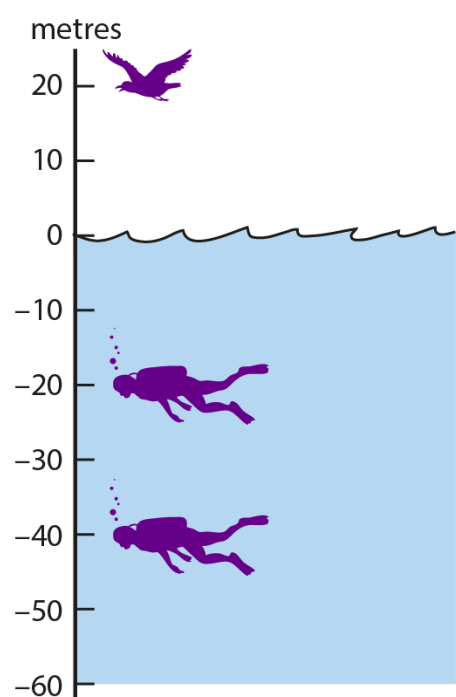
- *'Look at the temperatures at the South Pole in July. Explain why  $-63^{\circ}\text{C}$  is colder than  $-55^{\circ}\text{C}$ .'*

- 'Complete the table below.'

|   | South Pole | London |
|---|------------|--------|
| <b>Coldest minimum temperature (°C)</b>     |            |        |
| <b>Month of coldest minimum temperature</b> |            |        |
| <b>Warmest maximum temperature (°C)</b>     |            |        |
| <b>Month of warmest maximum temperature</b> |            |        |

Generalisation:

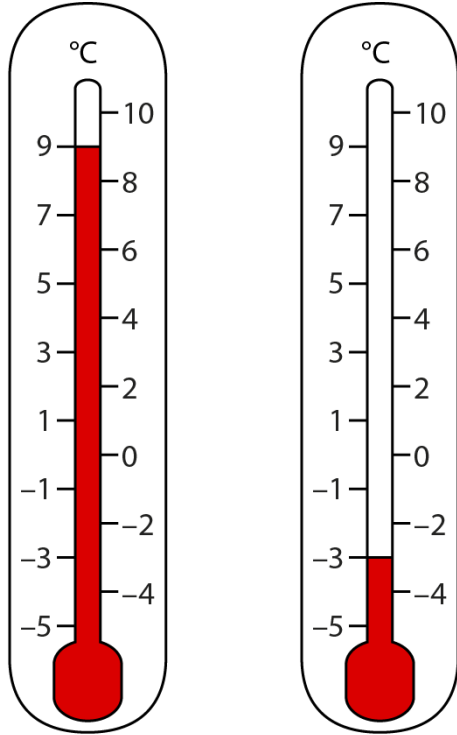


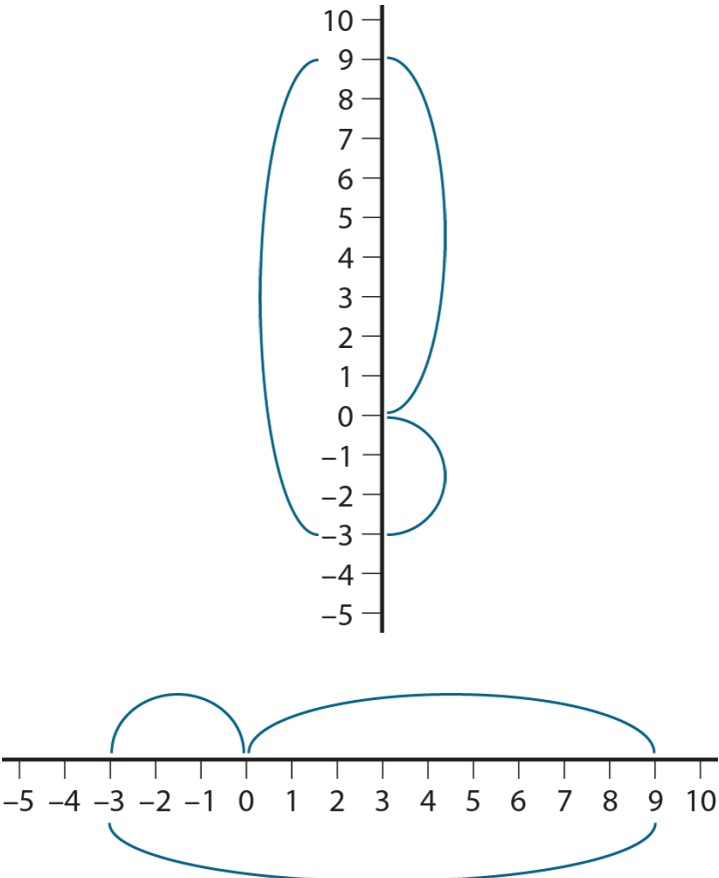
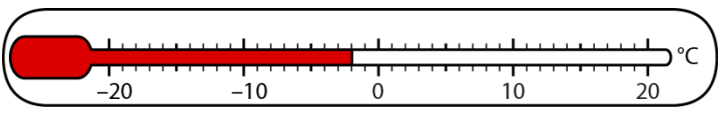
|  |   | <p>Height/depth relative to sea level:</p>   |  |                             |   |  |  |  |  |  |   |  |   |  |
|--|---|--|--|-----------------------------|---|--|--|--|--|--|---|--|---|--|
| 4:8  | Complete this teaching point by providing dòng não jīn problems, such as the examples opposite. | <p>Dòng não jīn:</p> <ul style="list-style-type: none"><li>'Decide whether each statement is true or false.'</li></ul> <table><tr><th></th><th>True (✓)<br/>or false<br/>(✗)</th></tr><tr><td>Today the temperature is <math>-2^{\circ}\text{C}</math> and yesterday the temperature was <math>-4^{\circ}\text{C}</math>, so today is colder than yesterday.</td><td></td></tr><tr><td>This morning the temperature was <math>-5^{\circ}\text{C}</math>. This afternoon the temperature was <math>-2^{\circ}\text{C}</math>. The temperature went up.</td><td></td></tr><tr><td>A submarine was at <math>-80\text{ m}</math>, then its depth changed to <math>-50\text{ m}</math>, so the submarine went down.</td><td></td></tr><tr><td>In a building, floor <math>-3</math> is above floor <math>-4</math>.</td><td></td></tr><tr><td>I was on floor <math>-2</math> of a building. I had to go down the stairs to get to floor <math>-1</math>.</td><td></td></tr></table> <ul style="list-style-type: none"><li>Explain how you could order the following numbers.</li></ul> <p>+18      -6.5      3      0.02      -15      3</p> |  | True (✓)<br>or false<br>(✗) | Today the temperature is $-2^{\circ}\text{C}$ and yesterday the temperature was $-4^{\circ}\text{C}$ , so today is colder than yesterday. |  | This morning the temperature was $-5^{\circ}\text{C}$ . This afternoon the temperature was $-2^{\circ}\text{C}$ . The temperature went up. |  | A submarine was at $-80\text{ m}$ , then its depth changed to $-50\text{ m}$ , so the submarine went down. |  | In a building, floor $-3$ is above floor $-4$ . |  | I was on floor $-2$ of a building. I had to go down the stairs to get to floor $-1$ . |  |
|  | True (✓)<br>or false<br>(✗)   |  |  |                             |   |  |  |  |  |  |   |  |   |  |
| Today the temperature is $-2^{\circ}\text{C}$ and yesterday the temperature was $-4^{\circ}\text{C}$ , so today is colder than yesterday.  |   |  |  |                             |   |  |  |  |  |  |   |  |   |  |
| This morning the temperature was $-5^{\circ}\text{C}$ . This afternoon the temperature was $-2^{\circ}\text{C}$ . The temperature went up. |   |  |  |                             |   |  |  |  |  |  |   |  |   |  |
| A submarine was at $-80\text{ m}$ , then its depth changed to $-50\text{ m}$ , so the submarine went down.                                 |   |  |  |                             |   |  |  |  |  |  |   |  |   |  |
| In a building, floor $-3$ is above floor $-4$ .  |   |  |  |                             |   |  |  |  |  |  |   |  |   |  |
| I was on floor $-2$ of a building. I had to go down the stairs to get to floor $-1$ .  |   |  |  |                             |   |  |  |  |  |  |   |  |   |  |

**Teaching point 5:**

Knowledge of the positions of positive and negative numbers in the number system can be used to calculate intervals across zero.

**Steps in learning**

|            | <b>Guidance</b>  | <b>Representations</b>  |
|------------|--|---|
| <b>5:1</b> | <p>This teaching point builds on children's understanding of the relative positions of positive and negative numbers to calculate across zero. Calculation is initially supported by number lines, providing children with an opportunity to answer questions using a number line, but by the end of the teaching point children should be able to calculate without necessarily relying on this scaffold.</p> <p>Begin by looking at calculating the difference between a positive and a negative number. Display two thermometers, one showing a positive number and the other showing a negative number (e.g. <math>9^{\circ}\text{C}</math> and <math>-3^{\circ}\text{C}</math>, as shown opposite). Recap what the images represent and how we know which temperature is warmer, referring to earlier generalised statements if needed.</p> <p>Then model how to find the difference between the two temperatures, using a number line. Initially use a reduction story, which will link to working back through zero on a number line: <i>'The temperature was <math>9^{\circ}\text{C}</math> in the day, then it dropped to <math>-3^{\circ}\text{C}</math> at night. What was the change in temperature?'</i> Show the calculation on both vertical and horizontal number lines. As usual, the number lines should be used to draw attention to the structure of the calculation (the fact that we 'step' through zero) rather than as a tool for calculation.</p> | <p><i>'The temperature was <math>9^{\circ}\text{C}</math> in the day, then it dropped to <math>-3^{\circ}\text{C}</math> at night. What was the change in temperature?'</i></p>  <p style="text-align: center;">Day                      Night</p> |

|   |   |
|---|---|
|   | <p>Note that on the number lines opposite, lines rather than arrows are used to represent the 'jumps'; this is to avoid confusion for children at this early stage. For example, the change from <math>9^{\circ}\text{C}</math> to <math>-3^{\circ}\text{C}</math> is, technically, <math>-12^{\circ}\text{C}</math>, where convention dictates that the negative/minus symbol indicates the drop in temperature; if we were to work backward on the number line, the 'jumps' would then be <math>-9^{\circ}\text{C}</math> and <math>-3^{\circ}\text{C}</math>, and these two negative numbers would be added together to make <math>-12^{\circ}\text{C}</math>. Instead, arrow heads are omitted, and the length of the jumps are added together (<math>9^{\circ}\text{C} + 3^{\circ}\text{C}</math>); to reach the final correct answer, children must then refer back to the context to say the temperature <i>dropped</i> by <math>12^{\circ}\text{C}</math>.</p> <p>Work through several examples, varying the context and whether the interval is calculated <i>from</i> the negative or the positive number; for example:</p> <ul style="list-style-type: none"> <li>• 'Jo was on the <math>-2\text{nd}</math> floor of a building. He took the lift to the <math>4\text{th}</math> floor. How many floors did he go up?'</li> <li>• 'A penguin was on an iceberg at <math>3\text{ m}</math> relative to sea level. The penguin jumped into the sea, diving down to <math>-6\text{ m}</math> relative to sea level. What was the total distance down that the penguin went?'</li> </ul>  <p>Summary: The temperature dropped by <math>12^{\circ}\text{C}</math>.</p> |
| <p><b>5:2</b></p> <p>Build on previous work of addition as augmentation and subtraction as reduction and finding the difference, to ask further questions that include calculating intervals across zero. Children should also be comfortable with calculation within the negative numbers (as in questions 6 and 7 opposite).</p> <p>For each calculation, ask children to discuss their strategies, which may include the following approaches:</p> |  <ul style="list-style-type: none"> <li>• 'What temperature does the thermometer show?'</li> <li>• 'The temperature rises by <math>16^{\circ}\text{C}</math>. Mark the new temperature reading on the thermometer.'</li> <li>• 'The temperature falls from <math>14^{\circ}\text{C}</math> to <math>-4^{\circ}\text{C}</math>. By how many degrees does the temperature fall?'</li> <li>• 'The temperature rises from <math>-4^{\circ}\text{C}</math> by <math>10^{\circ}\text{C}</math>. What is the new temperature?'</li> <li>• 'The temperature falls from <math>10^{\circ}\text{C}</math> by <math>16^{\circ}\text{C}</math>. What is the new temperature?'</li> </ul>   |

|             | <ul style="list-style-type: none"><li>When calculating the difference between <math>14^{\circ}\text{C}</math> and <math>-4^{\circ}\text{C}</math> (question 3 opposite), we might use the strategy of adding the two 'distances' from zero (i.e. <math>14 + 4</math>).</li><li>When calculating the new temperature after a rise of <math>10^{\circ}\text{C}</math> from <math>-4^{\circ}\text{C}</math> (question 4 opposite) we might partition the 10 into 4 and 6 to cross zero.</li><li>When calculating the new temperature after a fall of <math>16^{\circ}\text{C}</math> from <math>10^{\circ}\text{C}</math> (question 5 opposite), we might partition the 16 into 10 and 6 to cross zero.</li></ul>   | <ul style="list-style-type: none"><li><i>The temperature falls from <math>-6^{\circ}\text{C}</math> by <math>3^{\circ}\text{C}</math>. What is the new temperature?</i></li><li><i>The temperature is <math>-9^{\circ}\text{C}</math>. By how much must it rise to reach <math>-2^{\circ}\text{C}</math>?</i></li></ul>   |                      |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
|-------------|--|---|----------------------|---------|----------|-------------------|-------------|--|--|------|-----------|--|-----|----------------------|------------|--|-----|-----|------------|--|------|----------------------|------------|-----|--|------|------------|-----|------|----------------------|
| 5:3         | <p>Provide children with practice in a variety of contexts, with a variety of question structures (again including calculations within the negative numbers as well as across zero), such as those shown opposite and below:</p> <ul style="list-style-type: none"><li><i>'A submarine is below the surface of the water at <math>-40\text{ m}</math>. It goes up <math>15\text{ m}</math>, then down <math>5\text{ m}</math>. Where is it now?'</i></li></ul> <p>Before using the bank-account example opposite, you may wish to spend some time discussing what each column means, including the implications of a negative balance. Encourage children to represent the activity of the bank account on a number line.</p> <p>To promote and assess depth of understanding, provide a dòng não jīn problem such as the one shown opposite. This links to the cardinal value of number (similar to <i>Teaching point 1</i>). Here we are again looking at relative values / changes.</p> | <p><i>'Mr Money has £150 in his bank account at the end of December. The table shows money he paid in and out, and the amount in the account over the next few weeks. Fill in the missing numbers.'</i></p> <table><tr><th>Date</th><th>Paid in</th><th>Paid out</th><th>Amount in account</th></tr><tr><td>31 December</td><td></td><td></td><td>£150</td></tr><tr><td>5 January</td><td></td><td>£30</td><td><input type="text"/></td></tr><tr><td>10 January</td><td></td><td>£50</td><td>£70</td></tr><tr><td>11 January</td><td></td><td>£100</td><td><input type="text"/></td></tr><tr><td>12 January</td><td>£10</td><td></td><td>−£20</td></tr><tr><td>16 January</td><td>£20</td><td>£130</td><td><input type="text"/></td></tr></table> | Date                 | Paid in | Paid out | Amount in account | 31 December |  |  | £150 | 5 January |  | £30 | <input type="text"/> | 10 January |  | £50 | £70 | 11 January |  | £100 | <input type="text"/> | 12 January | £10 |  | −£20 | 16 January | £20 | £130 | <input type="text"/> |
| Date        | Paid in  | Paid out  | Amount in account    |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
| 31 December |  |   | £150                 |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
| 5 January   |  | £30   | <input type="text"/> |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
| 10 January  |  | £50   | £70                  |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
| 11 January  |  | £100  | <input type="text"/> |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
| 12 January  | £10  |   | −£20                 |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |
| 16 January  | £20  | £130  | <input type="text"/> |         |          |                   |             |  |  |      |           |  |     |                      |            |  |     |     |            |  |      |                      |            |     |  |      |            |     |      |                      |

Dòng nǎo jīn

*'Amir set himself the target of reading five books every month. The table shows how he records his reading record above and below his target.'*

| Jan | Feb | Mar | Apr | May | Jun |
|-----|-----|-----|-----|-----|-----|
| 0   | -2  | 3   | 1   | -1  | 2   |

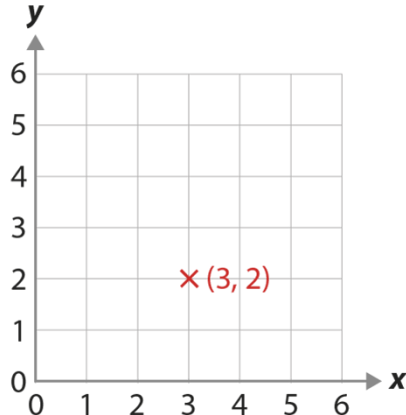
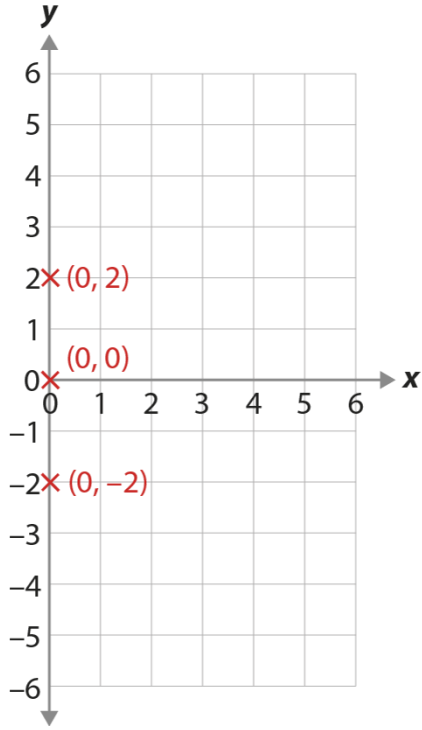
- *'How many books did Amir read in total?'*
- *'In which month did he read the least number of books?'*
- *'Between which months was the greatest difference in the number of books Amir read?'*

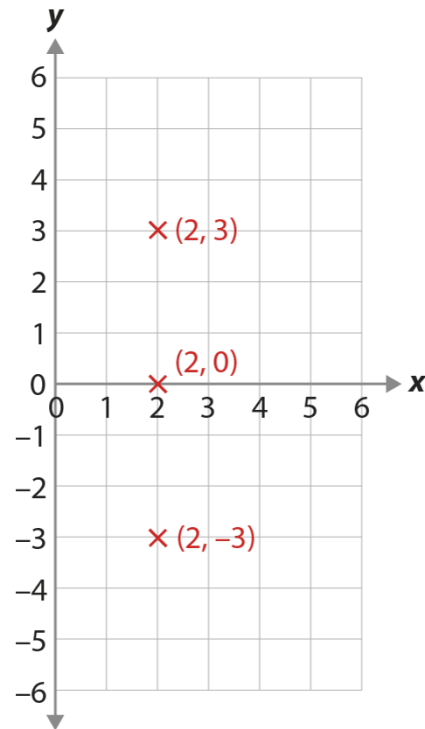


**Teaching point 6:**

Negative numbers are used in coordinate and graphing contexts.

**Steps in learning**

|            | Guidance  | Representations   |
|------------|---|---|
| <b>6:1</b> | <p>To complete this segment, explore negative numbers in the context of coordinate grids and more general graphing contexts.</p> <p>Children should already be familiar with plotting positive coordinates and expressing them in coordinate notation, for example <math>(3, 2)</math>. Review this now, and make links to the number line, drawing attention to the x-axis as a horizontal line and the y-axis as a vertical line.</p> <p>Then extend one of the lines to include negative numbers; here we extend the y-axis first. Draw children's attention to coordinates for which <math>x = 0</math> (i.e. the y-axis), for example <math>(0, 2)</math>, <math>(0, 0)</math> and <math>(0, -2)</math> etc., and make links to the vertical number line. Ask children what they notice about points above and below the x-axis (they should notice that points above the x-axis have a positive y-coordinate, while those below have a negative y-coordinate).</p> <p>Explore some coordinates with non-zero x-coordinates, for example, <math>(2, 3)</math>, <math>(2, 0)</math> and <math>(2, -3)</math>, working towards the following generalisation:</p> <ul style="list-style-type: none"> <li>• <b>'When the y-coordinate is negative, the point is positioned below the x-axis.'</b></li> <li>• <b>'When the y-coordinate is zero, the point is positioned on the x-axis.'</b></li> </ul> | <p>Reviewing the first quadrant:</p>  <p>Extending the y-axis:</p>  |

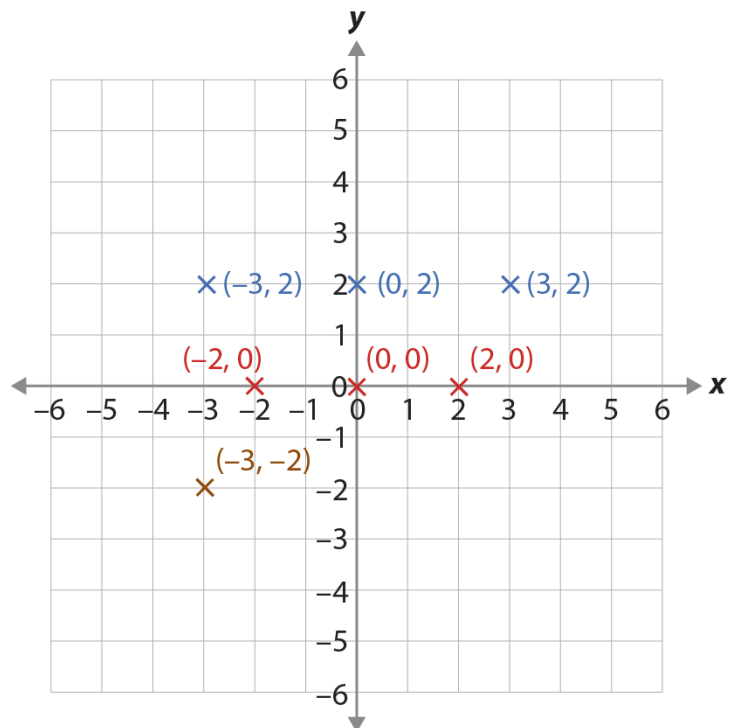
**6:2**

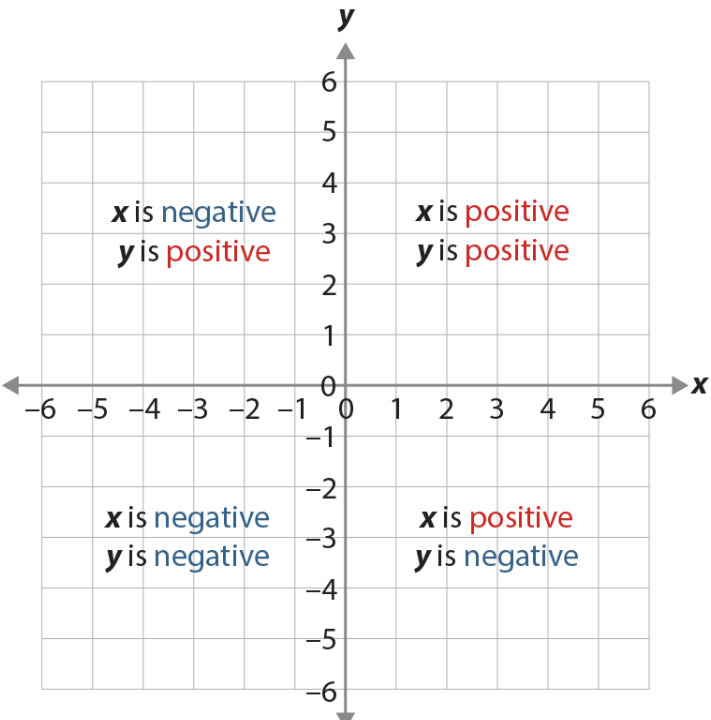
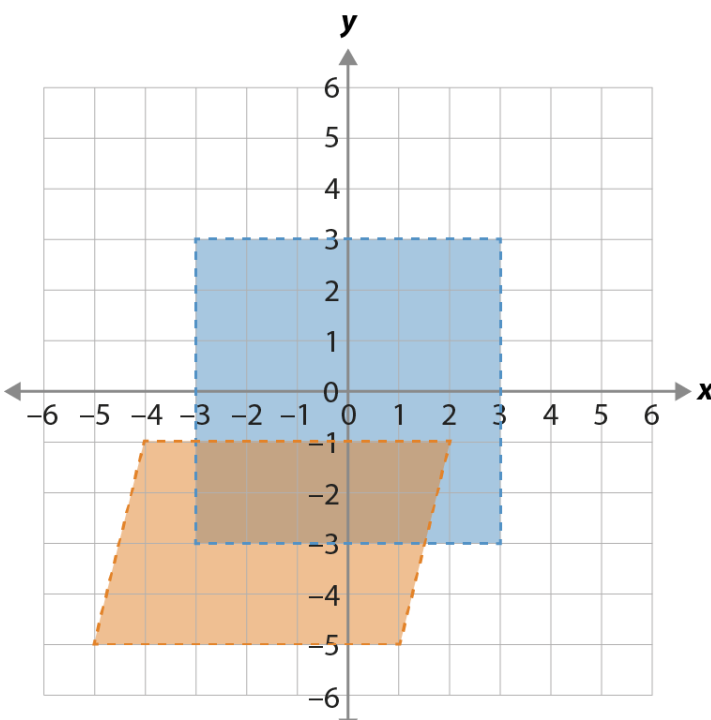
Now extend the other axis (here, the x-axis) to include negative numbers and ask children to reason about the location of the points when the x-coordinate is negative. In a similar way to step 6.1, focus on coordinates for which  $y = 0$  (i.e. at the x-axis), then explore coordinates with positive y-values. Work towards the generalisation:

- ***'When the x-coordinate is negative, the point is positioned to the left of the y-axis.'***
- ***'When the x-coordinate is zero, the point is positioned on the y-axis.'***

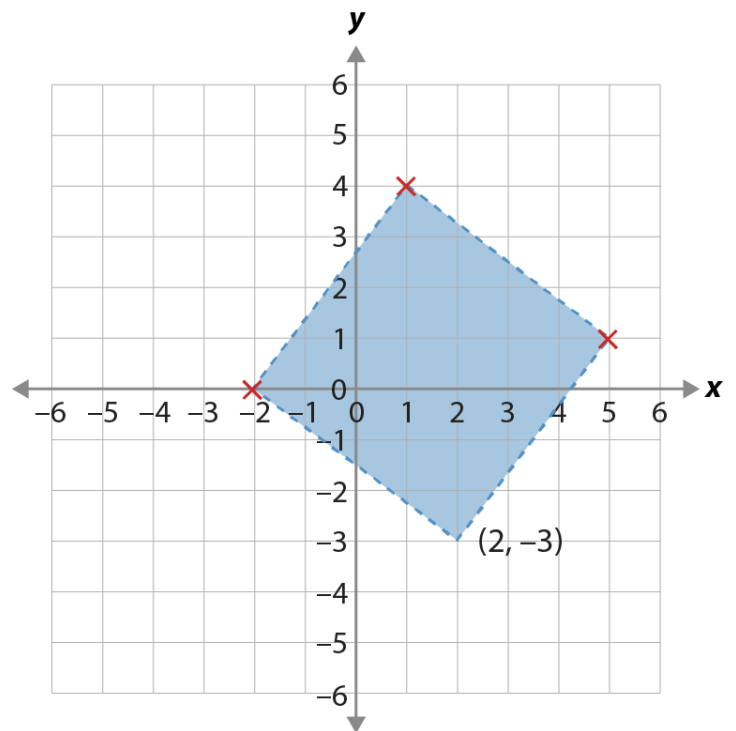
Finally, explore points in the remaining quadrant, for which both coordinates are negative (e.g.  $(-3, -2)$ ), and then summarise the properties of the four quadrants.

Extending the x-axis:



|                   |  |   |
|-------------------|--|---|
|                   |  | <p>Summary of quadrants:</p>    |
| <p><b>6:3</b></p> | <p>Children are now prepared for further teaching (Year 6; not covered in this spine) based on the coordinate grid, including:</p> <ul style="list-style-type: none"> <li>• drawing and labelling a pair of axes with equal scaling</li> <li>• drawing and labelling rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the four quadrants</li> <li>• predicting missing coordinates using the properties of shapes</li> <li>• working with coordinates expressed algebraically, for example: <ul style="list-style-type: none"> <li>• translating vertex <math>(a, b)</math> to <math>(a - 2, b + 3)</math></li> <li>• <math>(a, b)</math> and <math>(a + d, b + d)</math> being opposite vertices of a square of side-length <math>d</math>.</li> </ul> </li> </ul> | <ul style="list-style-type: none"> <li>• 'Draw a square of side-length 6, centred on the origin, <math>(0, 0)</math>.'</li> <li>• 'Draw a shape with vertices <math>(-4, -1)</math>, <math>(2, -1)</math>, <math>(-5, -5)</math> and <math>(1, -5)</math>. What type of shape is this?'</li> </ul>  |

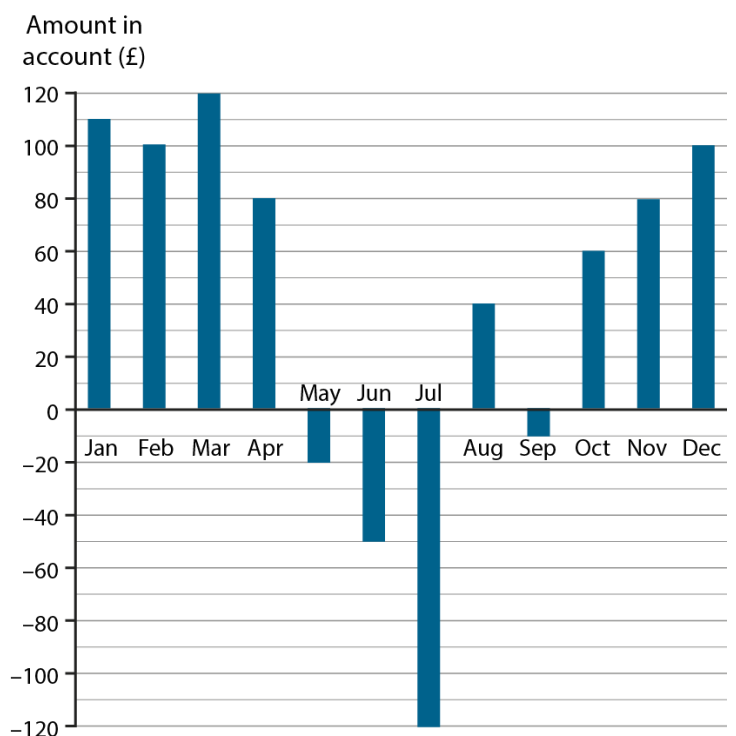
- The points  $(-2, 0)$ ,  $(1, 4)$  and  $(5, 1)$  are three of the four vertices of a square. What are the coordinates of the fourth vertex?



6:4

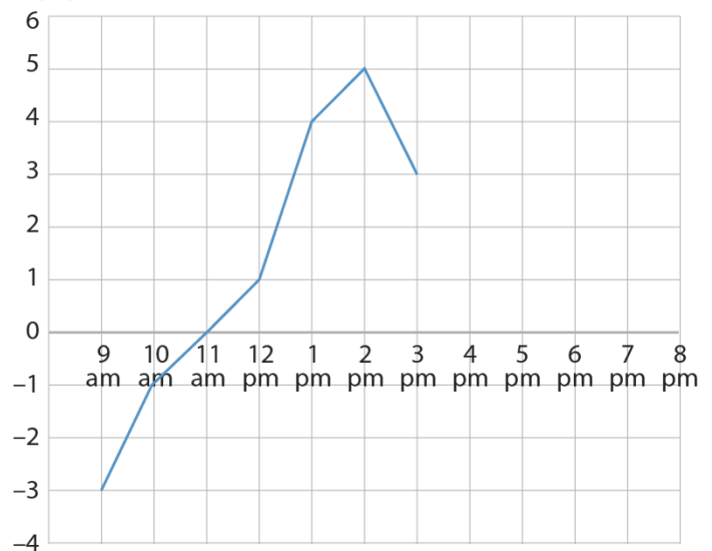
Finally, pose questions that require children to connect their work on number lines, coordinates and scales to their interpretation of other graphs that use negative numbers.

- The bar chart shows the amount of money in a bank account at the end of each month.'



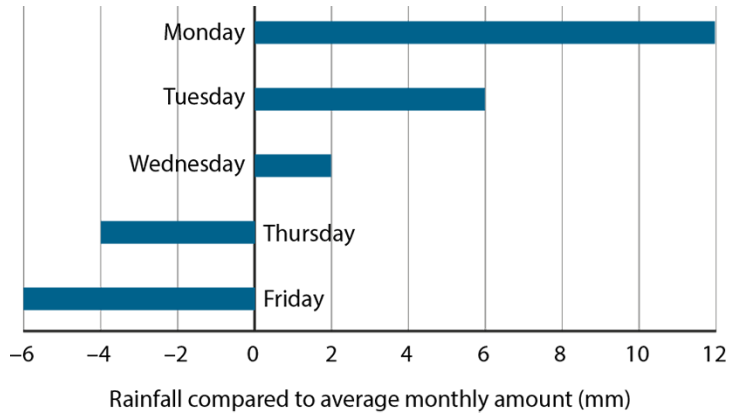
- 'What and when was the most amount of money in the account?'
- 'What and when was the least amount of money in the account?'
- 'What was the difference between the most and least amounts in the account?'
- 'How much money was paid into the account between July and August?'
- 'How much money was paid out of the account between August and September?'
- 'The graph shows the temperature in a school's nature garden during one day.'

Temperature  
(°C)



- 'What was the lowest temperature?'
- 'What was the highest temperature?'
- 'What was the difference between the lowest and highest temperatures?'
- 'By 8 pm the temperature had dropped to  $-1^{\circ}\text{C}$ . Plot this point on the graph. What was the change in temperature between 3 pm and 8 pm?'

- 'The bar chart shows the amount of rain that fell in the nature garden on different days of the week. The amounts are measured compared to the average monthly rainfall for that time of year.'



- 'What is the difference in rainfall between Wednesday and Thursday?'
- 'If the average monthly rainfall is 8 mm, how much rain fell on each day?'
- 'Why couldn't the average monthly rainfall for this period be 4 mm?'