MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.19 Securing mental strategies: calculation up to 999

Teacher guide | Year 3

Teaching point 1:

Known partitioning strategies for adding two-digit numbers within 100 can be extended to the mental addition of two-digit numbers that bridge 100, and addition of three-digit numbers.

Teaching point 2:

Transforming addition calculations into equivalent calculations can support efficient mental strategies.

Teaching point 3:

Subtraction calculations can be solved using a 'finding the difference' strategy; this can be thought of as 'adding on' to find a missing part.

Teaching point 4:

The order of addition and subtraction steps in a multi-step calculation can be chosen or manipulated such as to simplify the arithmetic.

Overview of learning

In this segment children will:

- extend known partitioning strategies and place-value understanding to calculations such as $87 + 56$ (bridging 100) and $370 + 260$, $307 + 206$ and $370 + 206$ (three-digit numbers with one zero digit)
- explore how addition calculations can be transformed to simplify the arithmetic, including an *'adjusting'* strategy (e.g. $35 + 49 = 35 + 50 - 1$) and a *'redistributing'* strategy (e.g. $35 + 49 = 34 + 50$)
- explore how subtraction calculations can be solved either by 'working forwards', using a *'finding the difference'* strategy (e.g. $43 - 39$ solved by adding on, i.e. $39 + \mathbf{1} = 40$, $40 + \mathbf{3} = 43$, $1 + 3 = 4$) or 'working backwards', by partitioning the subtrahend (e.g. $132 - 14 = 132 - 10 - 4$)
- be formally introduced to multi-step calculations involving both addition and subtraction, and explore how the order of operations can be manipulated to simplify the arithmetic.

This segment builds on:

- segments *1.15 Addition: two-digit and two-digit numbers* and *1.16 Subtraction: two-digit and two-digit numbers*, where children learnt to add and subtract two-digit numbers
- segments *1.17 Composition and calculation: 100 and bridging 100* and *1.18 Composition and calculation*: *three-digit numbers*, where children developed an understanding of three-digit numbers
- segment *1.12 Subtraction as difference.*

By this stage, children should have good place-value knowledge for numbers up to 999, solid understanding of addition and subtraction facts, and confidence in applying partitioning strategies for calculation with two-digit numbers. This segment explores some further strategies for adding and subtracting two- and three-digit numbers so that children can draw on flexible, mental strategies.

The subsequent two segments introduce formal written column algorithms for addition and subtraction (*1.20 Algorithms: column addition* and *1.21 Algorithms: column subtraction*). For children to be fluent in calculation, they need to be efficient, flexible and accurate. The purpose of this segment is to spend time addressing calculations for which column methods may not be the most efficient approach, for example to solve calculations like $499 + 499$ or $101 - 99$. The aim is not to learn a variety of methods 'by rote'. Rather, children should be encouraged to think flexibly and look for number relationships that can be used to simplify calculations; the ability to do this is a characteristic of growing mathematical confidence.

Throughout Year 3, and beyond, children should work towards being able to solve many of the calculations in this segment mentally. The role of the informal recordings here is to support the move to mental calculation and develop children's ability to make sensible decisions about fluent calculation, rather than to act as an alternative 'written method' running alongside column addition and column subtraction. However, some children will benefit from continuing to use jottings to record interim steps, to support the working memory. Where jottings are used, they should be quick and not overly arduous, for example:

```
132  –  14
        / \
      10   4
```

As usual, where Dienes or number lines are used to introduce a particular approach, their purpose is to reveal the strategy rather than to act as a tool for finding the answer; children should be encouraged to use known facts at each step of calculation.

In this segment, it is important to focus only on calculations that can easily be performed mentally. Children should not be encouraged to solve calculations such as *367 + 458* or *782 – 554* using these approaches, since these calculations are better suited to column methods. It is essential that teachers think carefully about the calculations they select as exemplars and practice throughout this segment. A useful guide is *'Do we want children to be able to do this calculation mentally?'* If the answer is no, then it shouldn't be included here. The examples included in the segment act as a further guide. This segment, however, does not aim to be exhaustive regarding all calculations where mental calculation should be encouraged, or indeed all strategies that could be used. Teachers should bear in mind that fluent calculation involves efficiency, accuracy and flexibility, and encourage these throughout their teaching.

Note that the finding the difference strategy introduced for subtraction calculations in *Teaching point 3* is a calculation *strategy* and is somewhat distinct from the subtraction *structure* of 'difference' as the comparison of two sets (introduced in segment *1.12*). The calculation *strategy* of 'finding the difference' can be used to solve subtractions involving a difference *structure* but can equally be applied to problems involving a partitioning or reduction structure.

Teaching point 1:

Known partitioning strategies for adding two-digit numbers within 100 can be extended to the mental addition of two-digit numbers that bridge 100, and addition of three-digit numbers.

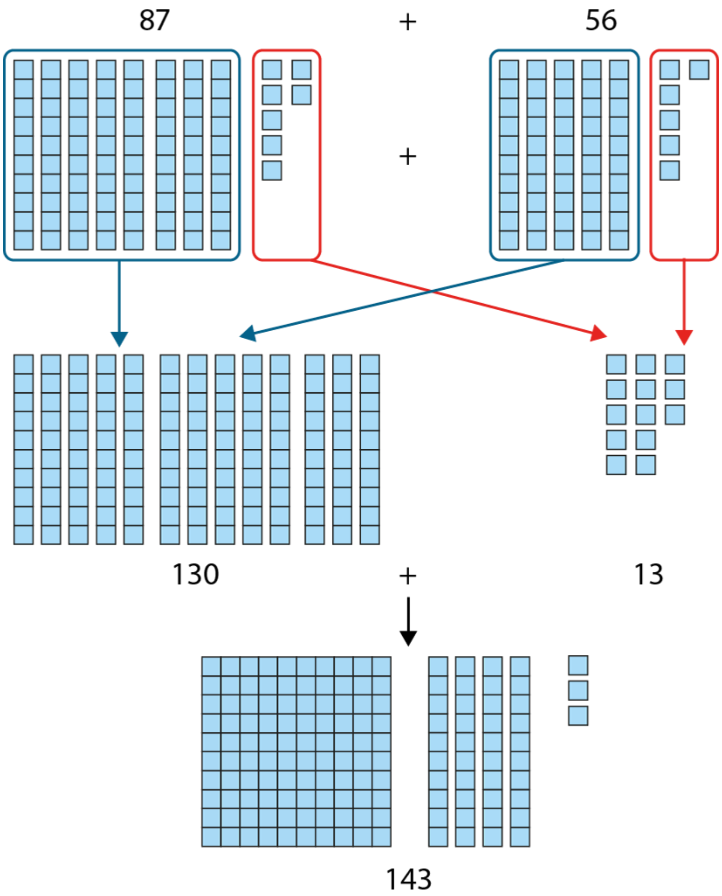
Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr>
<td>1:1</td>
<td>
In segment 1.15 Addition: two-digit and two-digit numbers, children learnt how to add two two-digit numbers, by partitioning either one (more efficient) or both addends, for numbers that sum to 100 or less. Briefly review these strategies before beginning this teaching point.
Here we extend application of the ‘partition both addends’ strategy to:

addition of two two-digit numbers whose sum is a three-digit number (e.g. 87 + 56) (step 1:1)
addition of two three-digit numbers (e.g. 370 + 260, 307 + 206 and 370 + 206) (step 1:2).

Introduce a two-digit addition calculation for which both a tens boundary and the hundreds boundary are bridged (e.g. 87 + 56). Work through the calculation with children, following the same steps as when there was no bridging of the hundreds boundary. Children should, by now, be very confident in the constituent steps:

80 + 50 = 130
adding multiples of ten across the hundreds boundary (segment 1.17 Composition and calculation: 100 and bridging 100, Teaching point 3)
7 + 6 = 13
bridging ten (segment 1.11 Addition and subtraction: bridging 10)
130 + 13 = 143
adding to the tens and ones, i.e. 130 + 10 + 3 (segment 1.18 Composition and calculation: three-

</td>
<td>
Dienes:


Jottings:
87 + 56 = 130 + 13 = 143
80 7   50 6
</td>
</tr>
</table>

	digit numbers, step *5:1*), or adding two two-digit numbers, i.e. *100* + ***30* + *13*** (segment *1.15*). Children need not use manipulatives to model the calculations, as they have already used them to gain a deep understanding of the constituent calculations, but opposite we show how Dienes can be used. At this stage, many children will still need to use pencil and paper jottings to record the constituent steps, but, as discussed in the overview, during Year 3 you should start to move children towards entirely mental calculation for examples such as these. The role of the jottings here is to support the move to mental calculation, but, even then, some children will need to jot down interim numbers to avoid working-memory overload. This does not detract from the fact that they are still using mental calculation strategies. Provide children with practice until they can confidently add two two-digit numbers that bridge 100. As children gain confidence they may not need to partition both addends, i.e.: • *87 + 50* adding a multiple of ten to a two-digit number (segment *1.17, Teaching point 4*) • *137 + 6* adding to the ones part (segment *1.18*, step *5:1*)	
1:2	Now extend this approach to the addition of two three-digit numbers. Keep to calculations for which each addend contains one zero digit, for example: 370 + 260 307 + 206 370 + 206	Method A: 370 + 260 = 37 tens + 26 tens = 63 tens = 630

	Note that the first two examples involve bridging tens/hundreds boundaries; calculations such as *210 + 130* were addressed in segment *1.18 Composition and calculation: three-digit numbers*, step *5:4*. Do not include calculations with three non-zero digits (e.g. *382 + 479*), since many of these are most efficiently added using column methods (special cases, such as *499 + 499*, are addressed in *Teaching point 2*). Begin with addition of two multiples of ten. Two possible strategies are presented: • method A: unitising in tens to transform to a two-digit calculation (37 tens + 26 tens) to which children can apply their partitioning strategies from segment *1.15 Addition: two-digit and two-digit numbers* • method B: partitioning the three-digit numbers in order to add the hundreds and add the tens, before recombining. These draw directly on known strategies and facts. If children need additional scaffolds, you can refer back to the manipulatives and representations in step *1:1*. Then consider calculations such as *307 + 206* and *370 + 206*, for which partitioning can again support efficient mental calculation. Provide some practice sequences as shown opposite.	Method B: $370 + 260 = 300 + 200 + 70 + 60$ $= 500 + 130$ $= 630$ Missing-number problems: *'Fill in the missing numbers.'* $29 + 13 = \square$ $\square = 440 + 370$ $290 + 130 = \square$ $\square = 540 + 370$ $608 + 307 = \square$ $\square = 570 + 106$ $607 + 308 = \square$ $\square = 506 + 170$
1:3	To complete this teaching point, provide practice for the calculation types covered, in the form of: • missing-number problems (equations or part–part–wholes) • real-life problems, including measures contexts; note that it is important for children to practise the	

strategies in response to a range of contexts (including augmentation and aggregation structures), recording the addition expression for each, but moving towards being able to perform the calculations mentally; example contexts are shown opposite and below:

- *'A sofa for the school library costs £350 and an armchair costs £160. How much money does the school need to buy them both?'* (aggregation)
- *'A farmer had 402 cows. During spring another 107 cows are born. How many cows does the farmer have now?'* (augmentation)

Missing-number problems:

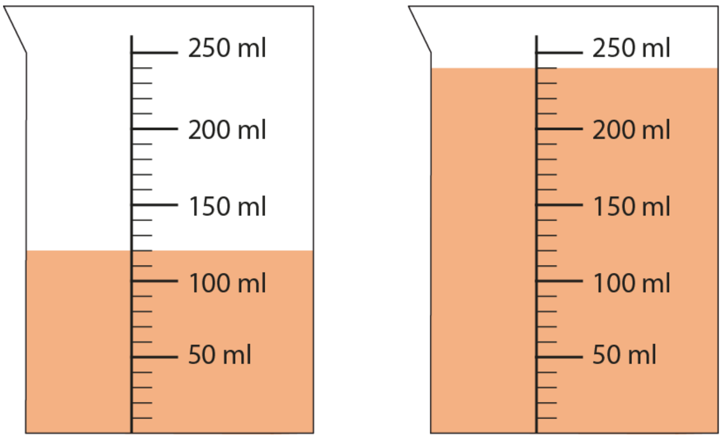
'Fill in the missing numbers.'

?	
67	44

56 + 83 = □	□ = 75 + 86
405 + 204 = □	□ = 107 + 707
250 + 430 = □	□ = 760 + 220
390 + 280 = □	□ = 270 + 640

Measures context (aggregation):

'How much water is there altogether?'



Dòng nǎo jīn:

672 + 297 = 969

'Use this equation to complete the following calculations.'

672 + 298 = □	662 + 298 = □
682 + 287 = □	572 + 198 = □
672 + □ = 968	□ + 297 = 970

Teaching point 2:

Transforming addition calculations into equivalent calculations can support efficient mental strategies.

Steps in learning

	Guidance	Representations
2:1	See below	See below

2:1 — Guidance

The partitioning strategy in the previous teaching point can be applied to perform many addition calculations mentally. However, it is not always the most efficient choice; for example, calculating *99 + 99* as *100 + 100 – 2* involves much simpler arithmetic than using a partitioning approach (*90 + 90 + 9 + 9*).

This teaching point explores strategies for transforming calculations and, crucially, develops children's ability to spot when these alternative strategies can be applied. Note that a two-digit calculation is used to introduce the strategies, but during this teaching point children will extend their understanding to three-digit calculations.

To introduce alternative addition strategies, present the three strategies, shown opposite, for calculating *35 + 49*, and ask children to evaluate them.

Children should notice that method A is the strategy that they have just been practising (*Teaching point 1*). Encourage them to explain that this strategy involves partitioning and adding.

As children evaluate method B, encourage them to notice that:

- 49 is close to 50
- *35 + 50* involves simpler arithmetic than *35 + 49*
- adding 50 is adding one too many, so we must then subtract one to find the correct answer.

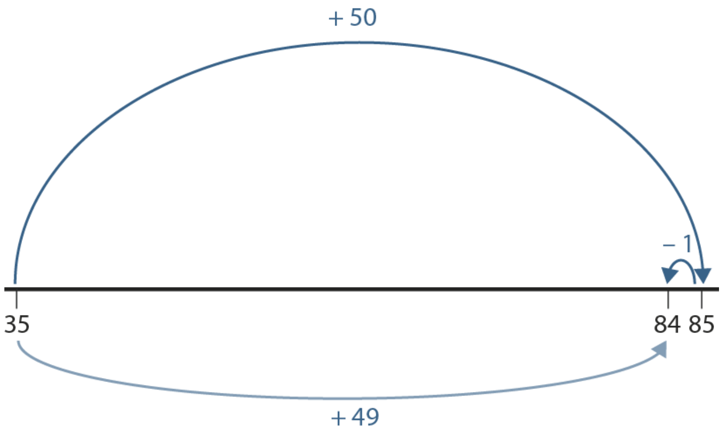
The number line reinforces the structure and can be used to support

2:1 — Representations

Method A:

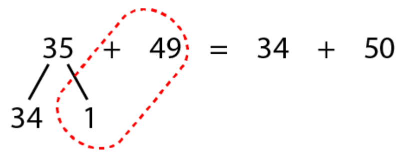
$$\begin{aligned} 35 + 49 &= 30 + 40 + 5 + 9 \\ &= 70 + 14 \\ &= 84 \end{aligned}$$

Method B:

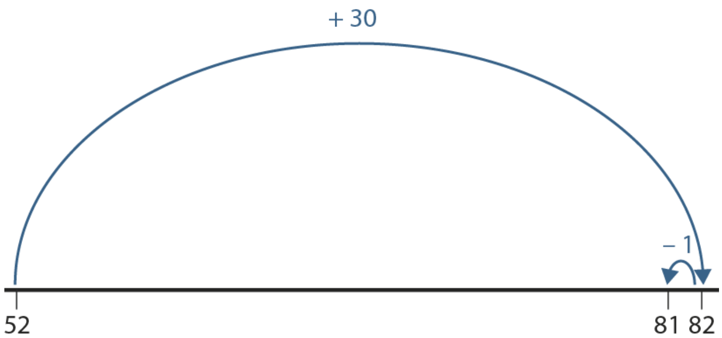


$$\begin{aligned} 35 + 49 &= 35 + 50 - 1 \\ &= 85 - 1 \\ &= 84 \end{aligned}$$

Method C:



$$\begin{aligned} 35 + 49 &= 34 + 50 \\ &= 84 \end{aligned}$$

	children in explaining why one is subtracted rather than added in the final step. Finally, ask children to use Dienes to model method C. Draw their attention to how the calculation has been transformed from *35 + 49* to *34 + 50*. Enacting this 'redistribution' (by moving a Dienes one cube across) helps children to recognise that the sum is unchanged. Ask them to discuss and explain why this redistribution simplifies the problem. The aim of this step in learning is for children to recognise that different strategies can be used to approach the same problem and to start getting familiar with some of these. Methods B and C are developed in the following steps (steps 2:2–2:5 and *2:6–2:11* respectively) until children are able to apply them appropriately and successfully. Note that children may propose other strategies too. Encourage and praise intelligent use of number relationships and other efficient strategies.	
2:2	Now work towards children being able to apply method B, which we will call the *'adjusting'* strategy. Look at a similar calculation to that shown for Method B in step *2:1*, for which one addend has nine ones (e.g. *52 + 29*). Again, model the calculation on a number line, writing the corresponding equation underneath.	Adjusting strategy (method B) – number line:  $52 + 29 = 52 + 30 - 1$ $= 82 - 1$ $= 81$

	Explain the steps clearly, with reference to the number line, using the following stem sentences: • ***'First we add: ___ plus ___ is equal to ___…'*** • ***'…then we adjust: ___ minus ___ is equal to ___.'*** Ask children *'Why do we need to adjust?'*, encouraging them to answer *'Because we have added too much.'* Then ask children to use the stem sentences to explain the steps to one another in pairs. Note that the number lines should be used to support understanding of the structure rather than as a tool for calculation; for the constituent steps, children should be using known mental facts/strategies. Give children practice solving a range of two-digit additions (within 100, and where one of the addends has nine ones), sketching their own number lines and using the stem sentences above for support as they write out the equations.	• *'First we add: fifty-two plus thirty is equal to eighty-two…'* • *'…then we adjust: eighty-two minus one is equal to eighty-one.'* • Summary: *'Fifty-two plus **twenty-nine** is equal to fifty-two plus **thirty minus one**.'*
2:3	Start to move children away from reliance on a number line; work with equations only, but scaffold the problems by showing linked calculations as shown opposite. A common difficulty that children experience is knowing which way to adjust in the final step (add one or subtract one). The number line supports children in realising that where the 'jump' has been too far, we need to subtract one, not add one. Once children are writing out their own jottings, a common mistake they make is to write, for example: $52 + 30 = 82 - 1 = 81$ ✗	

This recording is incorrect because the three terms in the equation don't all have the same value:

$$\begin{array}{ccccc} \underline{52 + 30} & = & \underline{82 - 1} & = & \underline{81} \ \times \\ \downarrow & & \downarrow & & \downarrow \\ 82 & & 81 & & 81 \end{array}$$

Work with any children who make this type of mistake, explaining why the equation is incorrect and reminding children of the meaning of the equals symbol.

Children should be encouraged to move away from the number line as a jotting, but should maintain it as a mental image to support their thinking when carrying out these types of calculations. Children may need explicit support to develop their powers of mental imagery, for example ask them to picture the number line with their eyes closed and say where particular 'jumps' will land.

Include examples where the first addend has nine ones (rather than the second) and vary the position of the equals symbol. As well as exemplar problems to present, an example of how children may work with the linked equations is shown opposite (example jottings). Encourage children to continue to use the stem sentences as they complete each calculation.

Finally progress to practice calculations without scaffolding.

Adjusting strategy (method B) – scaffolded missing-number problems:

'Fill in the missing numbers.'

$\mathbf{52 + 29 =} \boxed{}$ $\quad\quad$ $\mathbf{29 + 35 =} \boxed{}$

$52 + 30 = \boxed{}$ $\quad\quad$ $30 + 35 = \boxed{}$

$\mathbf{76 + 19 =} \boxed{}$ $\quad\quad$ $\mathbf{39 + 45 =} \boxed{}$

$76 + \boxed{} = \boxed{}$ $\quad\quad$ $\boxed{} + 45 = \boxed{}$

$\mathbf{27 + 39 =} \boxed{}$ $\quad\quad$ $\mathbf{19 + 73 =} \boxed{}$

$27 + \boxed{} = \boxed{}$ $\quad\quad$ $\boxed{} + 73 = \boxed{}$

Adjusting strategy (method B) – example jottings:

Step 1:

$\mathbf{76 + 19 =} \boxed{}$

$76 + \boxed{20} = \boxed{96}$

'First we add: seventy-six plus twenty is equal to ninety-six…'

Step 2:

$\mathbf{76 + 19 =} \boxed{95}$

$- 1$

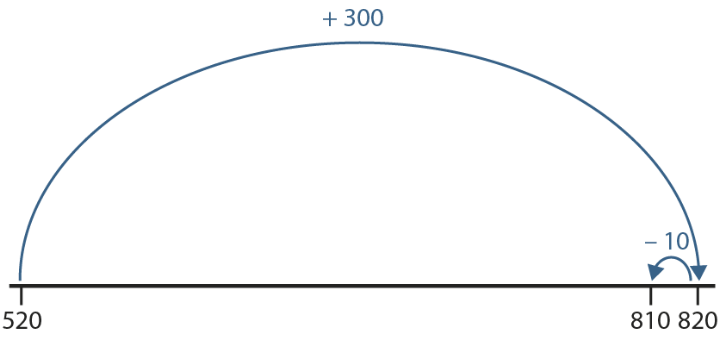
$76 + \boxed{20} = \boxed{96}$

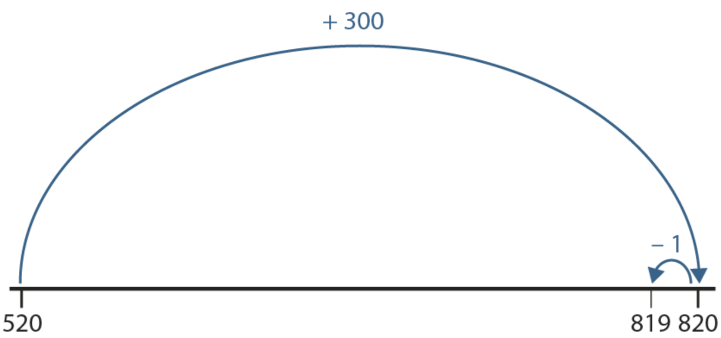
'…then we adjust: ninety-six minus one is equal to ninety-five.'

<table>
<tr>
<td>2:4</td>
<td>Now explore how the same strategy (adjusting, method B) can be used with three-digit addition calculations for which one of the addends is one or ten less than a whole hundred, for example:

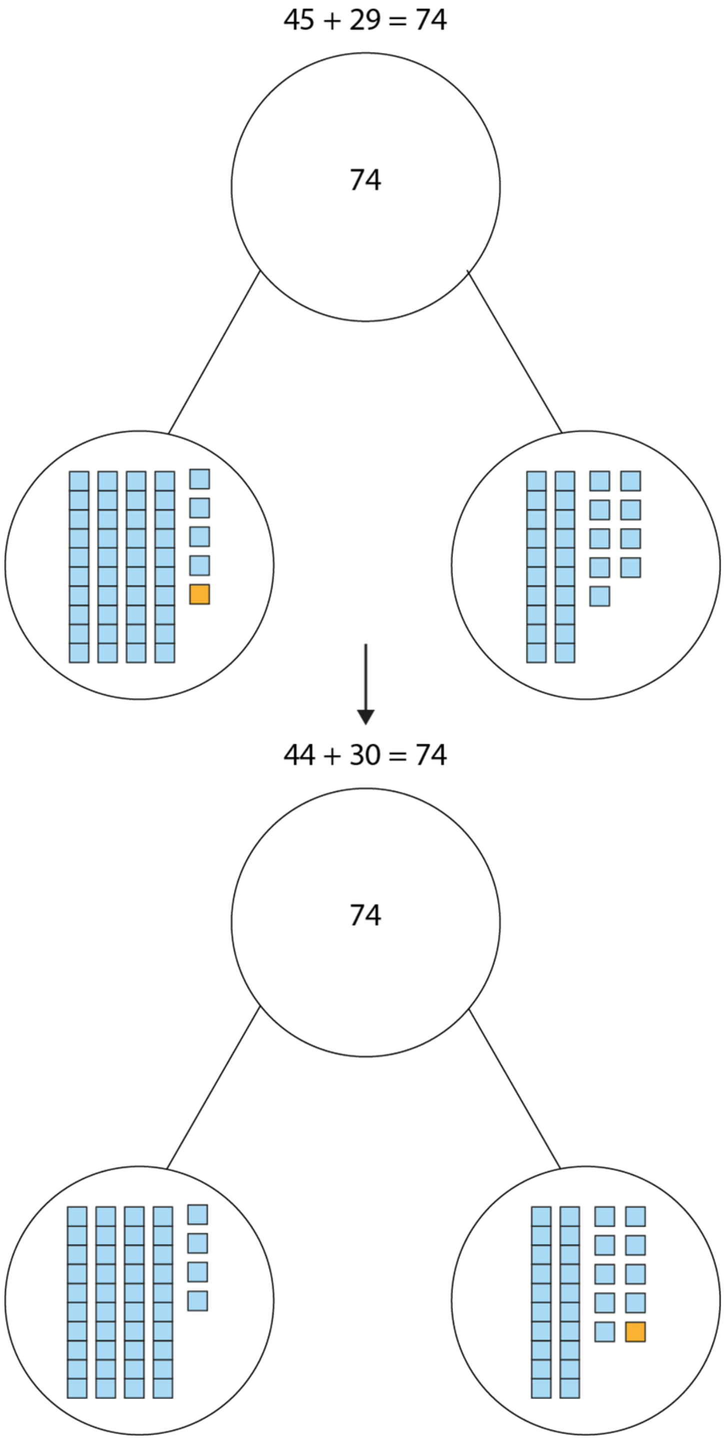
520 + 290

520 + 299

You can follow the same progression as in steps 2:2 and 2:3.</td>
<td>Adjusting strategy (method B) for three-digit numbers – example 1:
Number line

Linked equations
$\mathbf{520 + 290 =} \boxed{810}$
– 10
$520 + \boxed{300} = \boxed{820}$

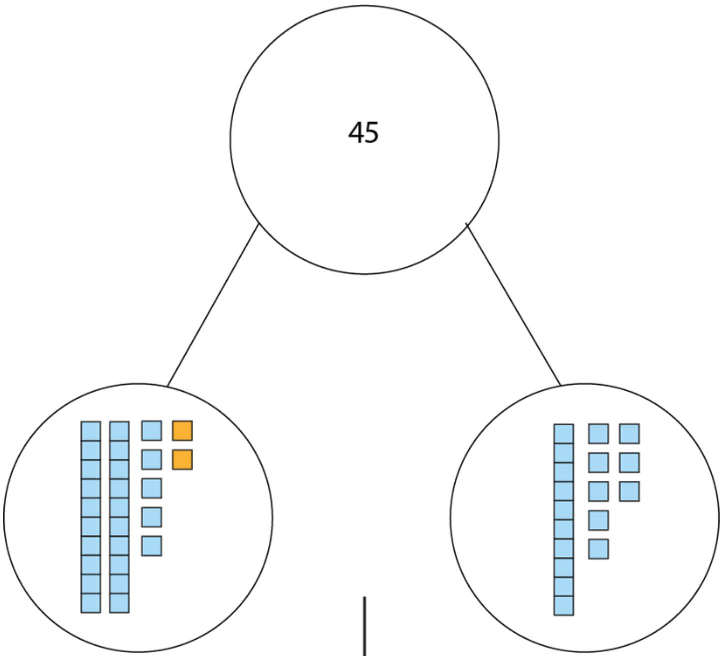
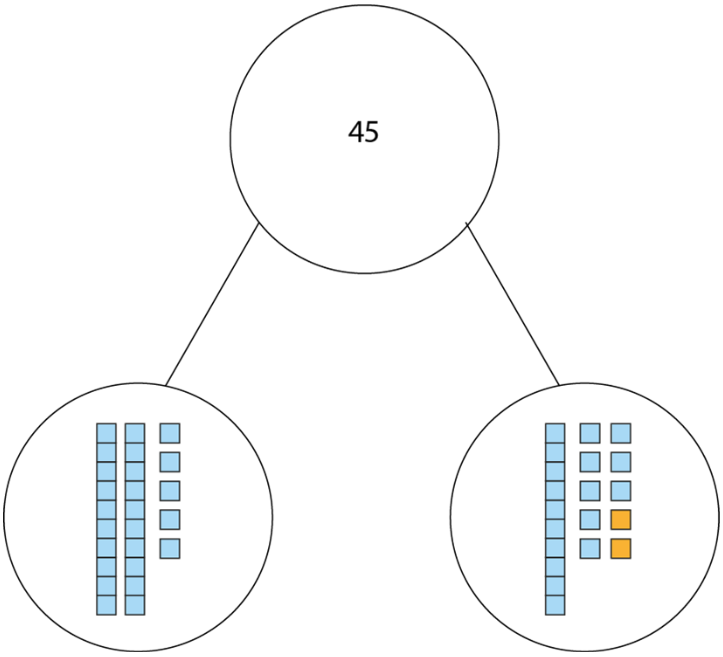
Adjusting strategy (method B) for three-digit numbers – example 2:
Number line

Linked equations
$\mathbf{520 + 299 =} \boxed{819}$
– 1
$520 + \boxed{300} = \boxed{820}$</td>
</tr>
</table>

2:5	Encourage children to explore this strategy in more detail by trying it in other situations, such as the examples shown opposite. For each example, draw attention to the fact that the amount subtracted is the total amount that the addends were adjusted by. Note that this strategy will be useful in Year 4, when children are adding prices (e.g. *£1.99* + *£3.99*). By now, children should be confident expressing this transformation using pairs of linked equations (although once they can do the thinking in their heads, there is no need to write down the equations). The number line is now less useful since more than one addend is being adjusted. To assess and promote depth of understanding, present a dòng nǎo jīn problem, such as that shown opposite, which encourages children to think about the adjusting strategy in reverse.	Adjusting strategy (method B): • Near-doubles – adjusting both addends **69 + 69 =** $\boxed{138}$ 70 + 70 = $\boxed{140}$ − 2 **290 + 290 =** $\square$ 300 + 300 = $\square$ $\square$ • Near triples – more than two addends **49 + 49 + 49 =** $\square$ 50 + 50 + 50 = $\square$ $\square$ • Near doubles – addends adjusted by different amounts **395 + 400 =** $\square$ 400 + 400 = $\square$ $\square$ • Near-doubles – measures context **2 m 99 cm + 2 m 99 cm =** $\square$ 3 m + 3 m = $\square$ $\square$ Dòng nǎo jīn: *'How many different ways can you find to complete the equation?'* $\square$ + $\square$ = 130 + 150 – 6 Example solution: 128 + 146 = 130 + 150 – 6

		(Note there are alternative ways to complete this, e.g. *228 + 46*, but encourage children to think about the adjusting strategy in reverse.)
2:6	Now return to method C, which we will refer to as the *'redistributing'* strategy; briefly review the example from step *2:1* again. There are certain similarities between this strategy and the adjusting strategy; the key difference is that, with redistribution, the total amount remains the same at all times, whereas, with adjustment, the total amount is increased to simplify the calculation and then decreased again. Whilst children need to understand these general approaches, the point here isn't to learn them both as 'rote' strategies, but rather to increase awareness of the power of transforming calculations in order to simplify them and give children some practice in doing so. Confident mathematicians will often look for ways to do this and the aim is for children to move beyond 'off-the-shelf' strategies. Explore the redistributing strategy with another two-digit calculation (e.g. *45 + 29*), again using Dienes. Use the part–part–whole model, as shown opposite, to draw attention to the fact that the 'whole', or sum, remains the same throughout. As usual, the Dienes should be used to draw attention to the structure, rather than as a tool for calculation. Using the Dienes, you could explore different redistributions of the sum, and then discuss which make the arithmetic easier (i.e. for *45 + 29* you could explore redistributing to *46 + 28* as well as to *44 + 30*). This form of exploration could be applied to the examples below as well. Then begin to move away from using the Dienes and part–part–whole	Redistribution strategy (method C) – part–part–whole model with Dienes: 

	models, with children practising representing pairs of equations, as shown opposite (the original equation and the transformed equation). Alternatively, children may write the calculations as a single equation: 45 + 29 = 44 + 30 = 74 Continue to emphasize the meaning of the equals symbol and the need for all three terms to have the same value.	Redistributing strategy (method C) – scaffolded missing-number problems: *'Fill in the missing numbers.'* **53 + 39 =** $\square$ **29 + 36 =** $\square$ 52 + 40 = $\square$ 30 + 35 = $\square$ **71 + 19 =** $\square$ **24 + 41 =** $\square$ 70 + $\square$ = $\square$ $\square$ + 40 = $\square$ **26 + 39 =** $\square$ **15 + 59 =** $\square$ 25 + $\square$ = $\square$ $\square$ + 60 = $\square$
2:7	Now look at how the redistributing strategy can be used with certain three-digit additions, for example where at least one of the addends is close to a whole hundred. You could look at the same examples as used in step *2:4*: 520 + 290 520 + 299 As in step *2:6*, begin by using Dienes and part–part–whole models, then progress to use of equations only to prepare for mental calculation. Once children have worked with the strategy for a range of examples and have a good sense of the underlying structure, encourage them to move to use of the generalised statement: ***'If one addend is increased by an amount and the other addend is decreased by the same amount, the sum remains the same.'*** The jottings opposite draw attention to this.	Redistributing strategy (method C) for three-digit numbers – example 1: **520 + 290 =** $\boxed{810}$ $\downarrow$ – 10 $\downarrow$ + 10 510 + 300 = $\boxed{810}$ Redistributing strategy (method C) for three-digit numbers – example 2: **520 + 299 =** $\boxed{819}$ $\downarrow$ – 1 $\downarrow$ + 1 519 + 300 = $\boxed{819}$

2:8	Provide practice with a range of examples. Encourage children to refer back to the generalisation for support and use the following stem sentence: ***'I have added ___ to this addend, so I need to subtract ___ from the other addend.'***	*'Use the first example in each set to help you write easier equivalent calculations.'* $21 + 39 = 20 + 40$ $22 + 39 = \square + \square$ $23 + 39 = \square + \square$ $24 + 39 = \square + \square$ $25 + 39 = \square + \square$ $470 + 90 = 460 + 100$ $470 + 190 = \square + \square$ $470 + 290 = \square + \square$ $470 + 390 = \square + \square$ $470 + 490 = \square + \square$ *'Fill in the missing numbers.'* **3 m 60 cm + 2 m 90 cm =** $\square$ 3 m 50 cm + $\square$ = $\square$

2:9	Once children are secure with redistributing one or one ten, extend to redistribution of two or two tens. Work through a calculation such as *27 + 18*, returning to the use of Dienes and part–part–whole diagrams. Draw attention to the fact that this time we need to add two to make a multiple of ten. Then ask children to use the generalisation and stem sentences to identify what we need to subtract from the other addend to keep the sum the same. (Note: an alternative redistribution would give *27 + 18 = 30 + 15*.) As before, begin to move away from using the Dienes and part–part–whole models, with children practising representing pairs of equations, as shown opposite.	Redistributing strategy (method C) – redistributing two ones: $\underbrace{27}_{25\quad 2} + 18 = 25 + 20$ 27 + 18 = 45  25 + 20 = 45 

		Redistributing two ones or two tens – scaffolded missing-number problems: *'Fill in the missing numbers.'* **34 + 18 =** $\square$ $\square$ + 20 = $\square$ **450 + 380 =** $\square$ $\square$ + 400 = $\square$ **27 + 28 =** $\square$ $\square$ + $\square$ = $\square$ **270 + 480 =** $\square$ $\square$ + $\square$ = $\square$
2:10	You can then further extend to redistributing three ones/tens, for example: 37 + 45 = 40 + 42 = 82 470 + 350 = 500 + 320 = 820	
2:11	Also give children opportunities to explore the flexibility of this strategy. The scaffolded example problems opposite prompt children to consider what happens depending on which 'direction' the redistribution occurs in (i.e. addend A to addend B, or vice versa).	Redistributing strategy (method C) – encouraging flexibility: *'Fill in the missing numbers.'* 68 + 85 = $\square$ + 90 68 + 85 = $\square$ + 80 68 + 85 = 70 + $\square$ 68 + 85 = 60 + $\square$
2:12	To compete this teaching point, provide varied practice to consolidate the use of the two strategies (adjusting and redistributing), including: • missing-number problems (now unscaffolded) • real-life problems including measures contexts, as shown below and opposite:	

- *'A bottle of juice contains 290 ml. How much juice is there in two bottles? How much is there in three bottles?'*
  (aggregation)
- *'Stephanie had 39 pence. Then she saved another 22 pence. How much does she have now?'*
  (augmentation)

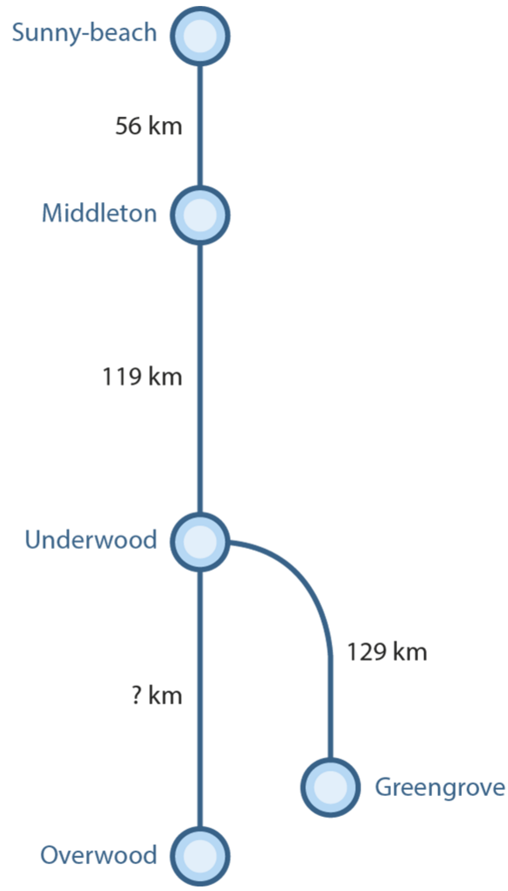
Note that, for many problems, there will be several different efficient strategies.

To further deepen understanding, present dòng nǎo jīn problems such as those shown opposite and below:

- *'Show three different ways to solve 398 + 495. Which is your preferred strategy? Why?'*

Measures context:

'The diagram shows the distances between stations on a railway route.'



- *'How far is it from Sunny-beach to Underwood?'*
- *'How far is it from Middleton to Greengrove?'*
- *'The journey from Underwood to Overwood is 44 km longer than from Underwood to Greengrove. How far is it from Underwood to Overwood?'*

Dòng nǎo jīn:

'Fill in the missing numbers.'

$99 + 9 + 3 = \boxed{}$

$48 + 19 + 4 = \boxed{}$

$98 + 8 + \boxed{} = 111$

Teaching point 3

Subtraction calculations can be solved using a ‘finding the difference’ strategy; this can be thought of as ‘adding on’ to find a missing part.

Steps in learning

3:1

Guidance

In segment *1:15 Addition: two-digit and two-digit numbers*, children learnt to subtract one two-digit number from another by partitioning the subtrahend and taking away both parts on a number line either *without* bridging a multiple of ten (e.g. *78 – 24 = 78 – 20 – 4*) or *with* (e.g. *44 – 16 = 44 – 10 – 6)*. Now, using a number line to support understanding of the structure, extend the calculation *without* bridging (*78 – 24*) to related three-digit examples:

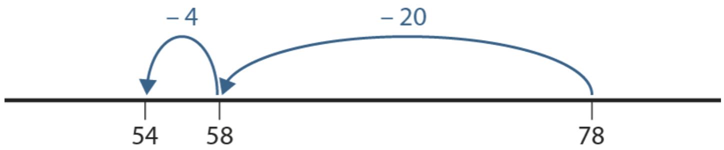
- *178 – 24*
  278 – 24
  378 – 24
  You can either work directly from the minuend as shown in the number-line example opposite (i.e. *178 – 20 – 4*) or partition and use the two-digit calculation/number line (i.e. *100 + 78 – 24*).
- *780 – 240*
  Again, you can work directly from the minuend (i.e. *780 – 200 – 40*) or unitise and use the two-digit calculation/number line (i.e. *78 tens – 24 tens*).

As before, the aim here is to make sure that children do not become reliant on written methods to perform these calculations. The number line should be used to help children to understand the strategies, but ultimately these calculations should be performed mentally. When children are ready,

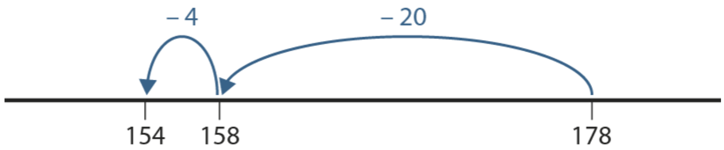
Representations

Partitioning the subtrahend (without bridging):

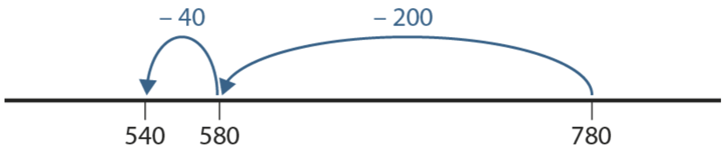
78 – 24



178 – 24



780 – 240



Missing-number problems:

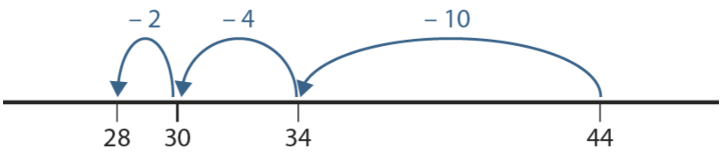
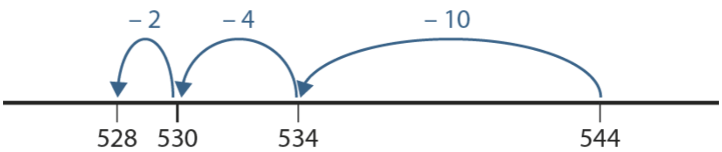
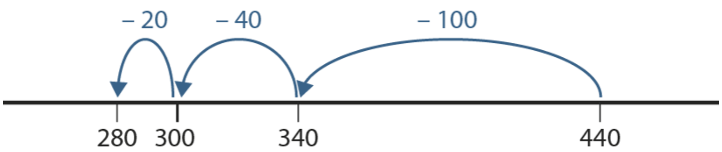
‘Fill in the missing numbers.’

85 – 13 = $\square$ 850 – 130 = $\square$

185 – 13 = $\square$

485 – 13 = $\square$

785 – 13 = $\square$

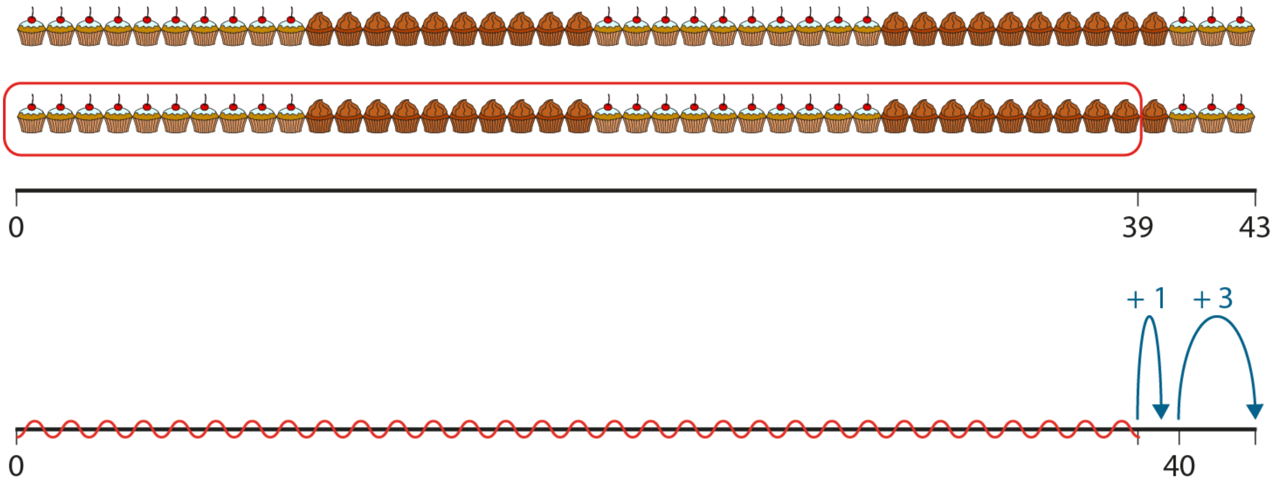
<table>
<tr><td></td><td>they can stop drawing the number line (or referring to an actual number line).
Provide some sequences of missing-number problems for practice.</td><td></td></tr>
<tr><td>3:2</td><td>Then repeat for calculations based on two-digit subtractions that do bridge a multiple of ten (e.g. 44 – 16), extending as before, for example, to:
544 – 16
440 – 160
Note that calculations in this and the previous step are restricted to those that can efficiently be solved mentally. Avoid calculations such as 431 – 274 that are better suited to column subtraction (see segment 1.21 Algorithms: column subtraction).</td><td>Partitioning the subtrahend (with bridging):
44 – 16

544 – 16

440 – 160

Missing-number problems:
‘Fill in the missing numbers.’
76 – 38 = $\boxed{}$ 760 – 380 = $\boxed{}$
176 – 38 = $\boxed{}$
476 – 38 = $\boxed{}$
776 – 38 = $\boxed{}$</td></tr>
<tr><td>3:3</td><td colspan="2">In segment 1.12 Subtraction as difference, children learnt about difference as a structure of subtraction (the comparison of two sets or two measures). Now we introduce ‘finding the difference’ as a strategy to solve subtraction calculations. This strategy is particularly useful when the minuend and subtrahend are close together in value (e.g. 43 – 39). Note that this calculation strategy can be applied to all subtraction structures (reduction, partitioning or difference).</td></tr>
</table>

Use a story to introduce the strategy:

- *'I baked forty-three cakes for the school fair.'*
  Show a picture of the cakes (as below) and discuss how we can tell there are 43 (they are arranged into four groups of ten, and three more).
- *'I sold thirty-nine of the cakes during the school fair.'*
  Ask a child to draw round the cakes that were sold. Discuss as a class that *any* 39 cakes might have been sold. We can't say for sure which cakes were left, and whichever 39 were sold, there will still be the same number of cakes left; however, the easiest way to show the 39 sold cakes is to work from the left-hand side of the row.
- *'How many cakes do I have left?'*
  Now the 39 cakes are circled, discuss how many are left with reference to the picture.

Then ask the children how we might represent this story on a number line, so we don't have to draw out all 43 cakes. Align a number line below the pictorial representation to make a clear link between the two. Although, in time, we want to move children to thinking about the difference between 39 and 43, and mentally considering only this part of the number line, at this stage, drawing out a 0–43 number line, 'crossing out' the section up to 39, helps children make the link between how the whole (here 43) and the two parts (here 39 and 4) fit together.

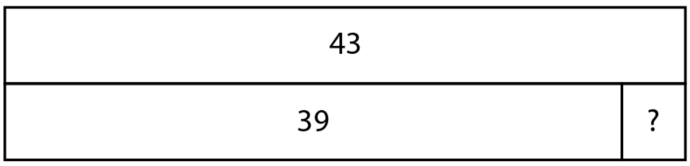
Work through several similar examples, until children see how they can use the bottom form of representation to subtract a large 'part' of the 'whole' (i.e. a large subtrahend relative to the minuend).



3:4

You can also represent the calculation on a bar model (though it is important to emphasise that the bar model is not the calculation strategy but rather a representation of the situation). This will help children make an association between the operation of subtraction and finding a missing part.

As an aside, it is worth noting the links between this finding the difference strategy and the 'subtracting from' strategy introduced in segment *1.11*



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Addition and subtraction: bridging 10. For the calculation 43 – 29, the latter involves partitioning the minuend (43 = 40 + 3), and subtracting 39 from 40:

43 – 39
40 3

40 – 39 = 1
1 + 3 = 4
so
43 – 39 = 4

It is worth being aware of this alternative strategy. To reiterate: the key idea is that wherever the 39 cakes are ‘taken’ from (be it the first 39, the last 39 or any other 39) there will still be the same number left.
</td>
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<tr>
<td>3:5</td>
<td>
Provide children with practice using this strategy for differences crossing a multiple of ten or crossing the hundreds boundary. Emphasize first adding to get to the nearest multiple of ten or 100, and then adding the remaining amount (this is sometimes called ‘shopkeeper addition’).

For now, choose examples where the value of the subtrahend is close to that of the minuend (numbers that are close together), as the strategy can be revealed more easily in this context. However, the strategy can later be extended to other calculations, for example, with intervening multiples of ten or 100, e.g. 611 – 298 (calculated using three jumps on the number line: + 2, + 300 and + 11).

At first, children will probably benefit from drawing number lines all the way from zero to the minuend as a scaffold. By the end of the sequence, encourage them to draw only the part of the number line showing the difference between the two numbers (i.e. from
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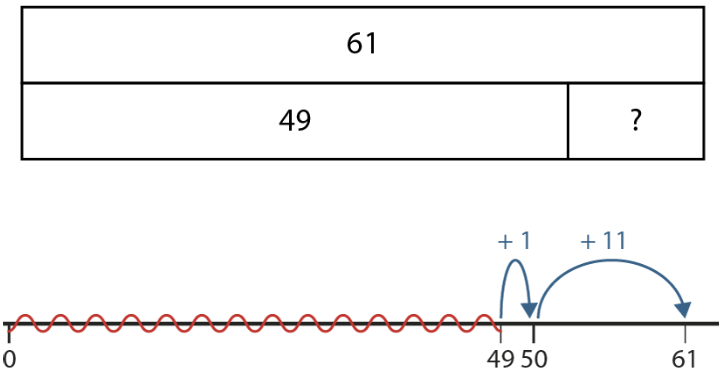
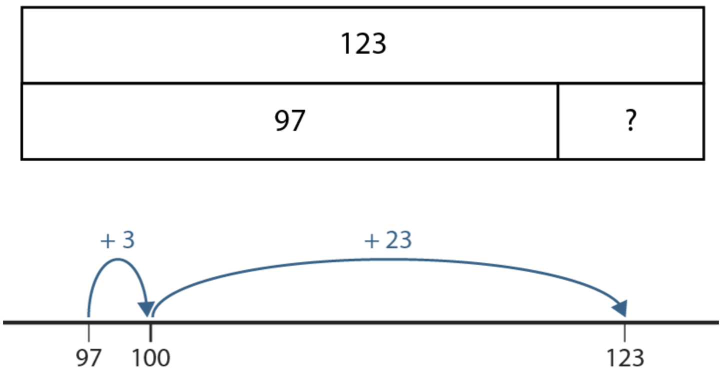
Using the full number line:


Using the number line to show only the difference:


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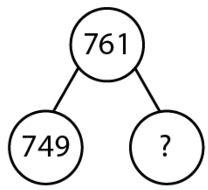
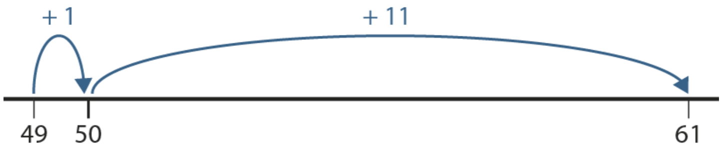
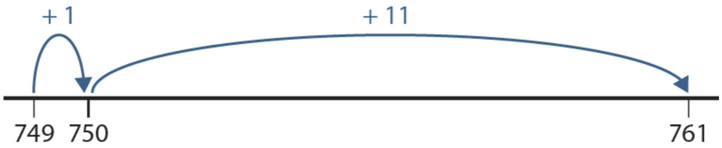
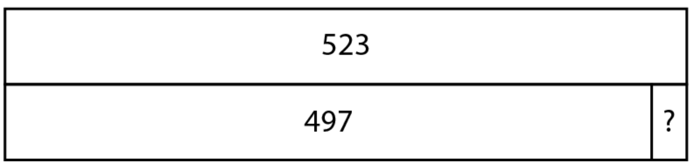
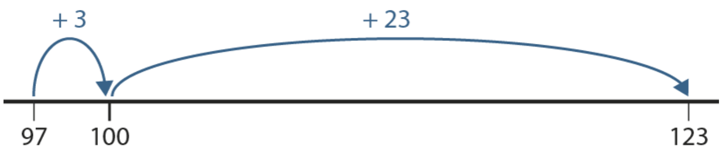
<td>the subtrahend to the minuend). Remember, the aim is eventually for children to calculate mentally, perhaps by visualising the difference on the number line.</td>
<td>Missing-number problems:
'Fill in the missing numbers.'
$\square = 82 - 59$ $103 - 99 = \square$
$\square = 95 - 82$ $115 - 88 = \square$
$\square = 73 - 65$</td>
</tr>
<tr>
<td>3:6</td>
<td>Now extend the finding the difference strategy to similar three-digit subtractions, both without bridging a multiple of 100 (e.g. 761 – 749) and with bridging (e.g. 523 – 497).

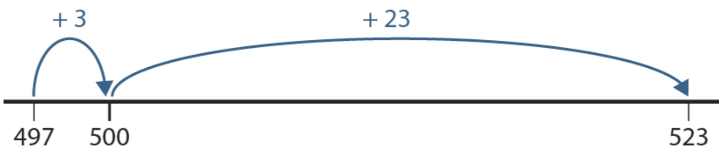
Note that the initial examples here have been chosen since they represent the same difference as the examples in step 3:5. Compare the examples (761 – 49 with 61 – 49, and 523 – 497 with 123 – 97) and encourage children to see that, even though the numbers are larger now, the calculations are no more complex.</td>
<td>Three-digit subtraction – without bridging a multiple of 100:






Three-digit subtraction – with bridging a multiple of 100:






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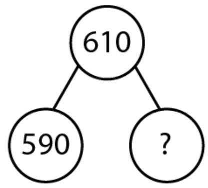
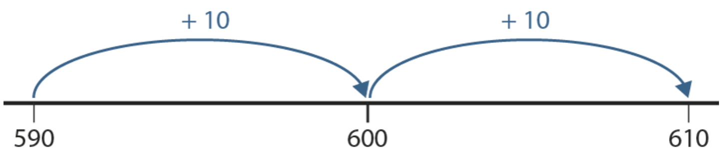
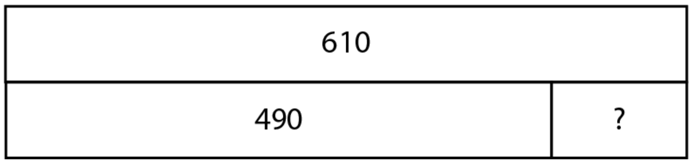
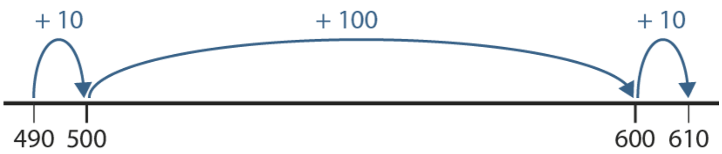
3:7	Now also extend the strategy to subtraction of three-digit multiples of ten within 1,000, for example: 610 – 590 610 – 490	   
3:8	Give children practice with the three-digit calculation types presented in steps *3:6* and *3:7* until they can confidently use the finding the difference strategy.	Missing-number problems: *'Fill in the missing numbers.'* $72 - 68 = \square$ $105 - 98 = \square$ $172 - 168 = \square$ $105 - 88 = \square$ $572 - 568 = \square$ $\square = 93 - 79$ $\square = 407 - 390$ $\square = 393 - 379$ $\square = 407 - 380$ $\square = 893 - 879$ $420 - 390 = \square$ $420 - 290 = \square$

3:9

Once children have mastered the application of the finding the difference strategy (adding on), it is important for them to learn to identify when this approach is an efficient choice. Present a range of subtraction calculations to work through together:

- some of which are most efficiently solved by working back from the minuend (partitioning the subtrahend as appropriate); examples should have a small subtrahend relative to the minuend (e.g. *132 – 14*)

and

- some of which are most efficiently solved by working forward from the subtrahend (finding the difference); as already discussed, examples should have a large subtrahend relative to the minuend (e.g. *132 – 125*) or include intervening multiples of ten or 100 (e.g. *611 – 298*).

Encourage children to consider which strategy is most efficient for each calculation, explaining their reasoning.

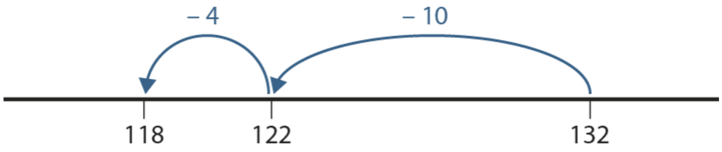
For both strategies, gradually support the transition away from drawing a number line to full mental calculation by encouraging children to visualise the number line instead; you may wish to 'act it out' at the front of the classroom with a child, for example, for *132 – 125*:

> **You:** *'First I need to get from one hundred and twenty-five to one hundred and thirty…'*
> (miming the jump by 'drawing' the curved line in the air)
>
> **Child:** *'That's five… And now I need to get from one hundred and thirty to one hundred and thirty-two.'*

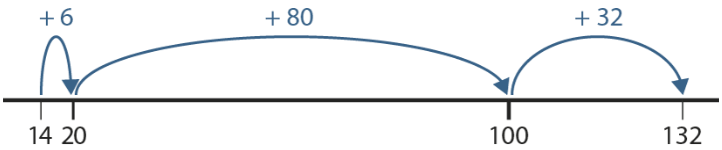
Small subtrahend:

132 – 14 = $\square$

- Partitioning the subtrahend (working back from the minuend) – *more* efficient



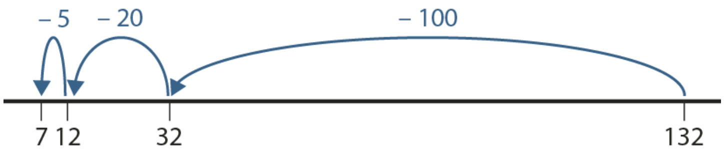
- Finding the difference (working forward from the subtrahend) – *less* efficient



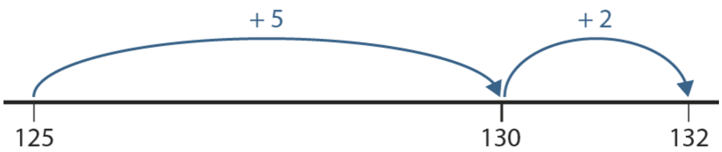
Large subtrahend:

132 – 125 = $\square$

- Partitioning the subtrahend (working back from the minuend) – *less* efficient



- Finding the difference (working forward from the subtrahend) – *more* efficient



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<td>(miming a smaller jump in the air, from the previous end point)
Child: 'That's two.'
You: 'So the difference is?'
Child: 'Five plus two. Seven.'</td>
<td></td>
</tr>
<tr>
<td>3:10</td>
<td>Provide a range of subtraction expressions and ask the children to sort them according to the most efficient strategy. Children may have different preferred strategies, which is fine as long as they are able to calculate efficiently and confidently.

It is worth noting that, even with the teaching broken down in this way, fluency in choosing between the strategies will need to be reinforced throughout Year 3; teaching it once and then leaving it is unlikely to result in mastery. Also bear in mind that, as for addition, depending on the numbers involved, there may be other efficient strategies.</td>
<td>'Sort the calculations according to the most efficient strategy.'
810 – 680
276 – 43
720 – 130
561 – 495
56 – 49
<table>
<tr><th>Working back / partitioning</th><th>Working forward / finding the difference</th></tr>
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<tr>
<td>3:11</td>
<td>Finish this teaching point by providing varied subtraction practice including:

missing-number problems (equations and part–part–wholes)
real-life problems, including measures contexts, for example:

'There are 420 children in a school. 110 are going on a school trip. How many are not going on the trip?'
(partitioning)
'A bottle of juice contained 750 ml. I drank 230 ml of the juice. How much is left?'
(reduction)
'I am hanging bunting along the top of the fence for the school fair. The fence is 75 m long. I have 58 m of bunting. How much more bunting do I need?'
(difference)

</td>
<td>Missing-number problems:
'Fill in the missing numbers.'
<table>
<tr><td colspan="2">87</td></tr>
<tr><td>59</td><td>?</td></tr>
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<table>
<tr><td colspan="2">57</td></tr>
<tr><td>?</td><td>24</td></tr>
</table>
$431 - 25 = \boxed{}$ $530 - 470 = \boxed{}$
$162 - 148 = \boxed{}$ $740 - 150 = \boxed{}$
$405 - 398 = \boxed{}$ $610 - 30 = \boxed{}$

Dòng nǎo jīn:
'Fill in the missing numbers.'
$140 + \boxed{} = 820$ $136 = \boxed{} + 19$</td>
</tr>
</table>

	Note that the labelling of these examples (partitioning, reduction and difference) refers to the subtraction *structures* exemplified, not the most appropriate *strategies* for solving them. As the ultimate aim is mental calculation, think carefully about whether or not you want to ask all children to continue to draw a number line. Of course, if a child opts to solve the problems mentally, but is making lots of mistakes, you will probably want to direct them back to using a number line for a while. To provide further challenge and depth, present dòng nǎo jīn problems such as those shown opposite and below: • *'I have collected 136 stickers. Sam has collected 22 fewer than me. How many stickers do we have altogether?'*	

Teaching point 4

The order of addition and subtraction steps in a multi-step calculation can be chosen or manipulated such as to simplify the arithmetic.

Steps in learning

	Guidance	Representations
4:1	The aim of this teaching point is to enable children to manipulate two-step problems so that they can perform the addition/subtraction steps in the most efficient order. In this step, the focus is on demonstrating that the order of the addition and subtraction operations has no bearing on the answer. Steps *4:2* and *4:3* then move on to show that, depending on the numbers involved, one order may be easier/more efficient than the other. Begin by presenting two related stories, for example: • *'Nina had 55 stickers. She gave 12 away. Then she got 4 more. How many stickers does she have now?'* • *'Rudy had 55 stickers. He got 4 more. Then he gave 12 away. How many stickers does he have now?'* For each scenario, ask the children to imagine it, then, as a class, write an expression to represent it: 55 – 12 + 4 55 + 4 – 12 Ask the children whether they think Nina and Rudy end up with the same number of stickers; then work through each calculation, adding/subtracting in the order presented in the corresponding story, to prove that the final number of stickers is the same in each case. Then work through another similar	*'Nina had 55 stickers. She gave 12 away. Then she got 4 more. How many stickers does she have now?'* $55 - 12 + 4 = \boxed{47}$ $55 - 12 = 43$ $43 + 4 = 47$ *'Rudy had 55 stickers. He got 4 more. Then he gave 12 away. How many stickers does he have now?'* $55 + 4 - 12 = \boxed{47}$ $55 + 4 = 59$ $59 - 12 = 47$

problem, again drawing attention to the fact that the order in which the addition and subtraction operations are performed has no effect on the outcome.

Work towards the generalised statement: ***'For calculations that involve both addition and subtraction steps, we can add then subtract, or subtract then add; the final answer is the same.'***

4:2

Once children are secure in their understanding that the order of operations does not affect the answer, present a new example to highlight that one order may be more efficient than the other, for example:

45 + 17 – 5

45 – 5 + 17

Ask the children to solve both calculations in the order written, and then ask them to explain which they thought was easiest. Encourage children to see that, in this case, subtracting then adding is probably more efficient, and gives the same answer, so if we are presented with *45 + 17 – 5*, we can choose to subtract the 5 from the 45 first.

Work through several pairs of calculations, asking children to spot which order is most efficient. Then present 'one-off' calculations, asking children to identify whether the presented order is efficient or whether it would be better to swap the order of operations.

Equivalent calculations:

45 + 17 – 5 = $\boxed{57}$

45 + 17 = 62

62 – 5 = 57

45 – 5 + 17 = $\boxed{57}$

45 – 5 = 40

40 + 17 = 57

Identifying the most efficient order of operations:

- *'Which order is best? Circle the most efficient choice for each example.'*

132 + 17 – 2	426 – 26 + 72
132 – 2 + 17	426 + 72 – 26

- *'For each calculation, decide whether it would be better to calculate in the order shown or to swap the addition and subtraction steps.'*

95 – 7 + 5	623 + 42 – 23
59 – 16 + 11	585 + 15 – 120

Keep the same	Swap the steps

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Now use a story to explore an example where the calculation can be simplified by not starting with the first term, for example:

'A baker had 265 buns. Then he made 84 more buns. Then he sold 83 buns. How many buns does he have now?
'A baker had 84 buns. Then he sold 83 buns. Then he baked 265 more buns. How many buns does he have now?'

Follow the same progression as in step 4:1, writing expressions to match each scenario, performing the calculations in the order presented and emphasizing that both answers are the same. Discuss how the calculation is simpler when we calculate 84 – 83 first:
$265 + \underbrace{84 - 83}_{1} = 266$
Then work through another similar problem, again drawing attention to the fact that the order in which the addition and subtraction operations are performed has no effect on the outcome.
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<td>4:4</td>
<td>
To complete this teaching point, provide varied practice including:

missing-number problems
real-life problems, including measures contexts, for example:

'A driver had 83 litres of diesel in her lorry. She added 29 litres more before she started work. Then she used 33 litres of diesel during the day. How much is left at the end of the day?'
'A shop had 330 toy cars. It sold 56 cars. Then 36 more cars were delivered. How many toy cars are there now?'

</td>
<td>
Missing-number problems:
'Fill in the missing numbers.'
$79 + 24 - 19 = \square$ $\square = 343 - 25 + 7$
$165 + 18 - 15 = \square$ $\square = 640 + 56 - 16$
$815 + 15 - 20 = \square$ $\square = 470 + 50 - 20$
</td>
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	In all cases, encourage children to look for the simplest approach. Make sure you include a variety of problems such that some are more easily solved in the order presented, while others can be simplified by rearranging the terms. It is also useful to include some cases where there is no obvious simpler way, to promote discussion. The key to success with these strategies, as with others presented in this segment, is being able to recognise when they are appropriate.	Dòng nǎo jīn: *'Fill in the missing numbers.'* $85 + \boxed{} - 5 = 90$ $165 + 14 - \boxed{} = 175$ $343 - 3 - \boxed{} = 300$