MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.15 Addition: two-digit and two-digit numbers

Teacher guide | Year 2

Teaching point 1:

Known strategies can be combined to add two multiples of ten to two single-digit numbers.

Teaching point 2:

Two two-digit numbers can be added by partitioning one or both of them into tens and ones.

Overview of learning

In this segment children will:

- apply ideas of commutativity and associativity to add together four numbers (two multiples of ten and two single-digit numbers)
- extend known strategies (including partitioning the addends into tens and ones, as well as commutativity and associativity) to the addition of two two-digit numbers.

This segment draws together several concepts/strategies from previous segments, and applies them to the addition of two two-digit numbers. For children to be successful in this segment, they must already have mastered:

- partitioning a given two-digit number into tens and ones (segments *1.9 Composition of numbers: 20–100* and *1.10 Composition of numbers: 11–19*)
- strategies for bridging ten (segment *1.11 Addition and subtraction: bridging 10*)
- adding a single-digit number to a two-digit number (segment *1.13 Addition and subtraction: two-digit and single-digit numbers*)
- adding two multiples of ten (segment *1.8 Composition of numbers: multiples of 10 up to 100*)
- adding multiples of ten to a two-digit number (segment *1.14 Addition and subtraction: two-digit numbers and multiples of ten*).

Children are not introduced to any new concepts in this segment; instead, they will combine these skills to explore the different ways that two two-digit numbers can be added together. As with previous segments, the examples chosen should reflect the different structures of addition, and questions should be presented using a variety of different contexts and measures (money, mass, volume, length, area, etc.).

Teaching point 1 explores the addition of two multiples of ten and two single-digit numbers (e.g. $40 + 20 + 5 + 3 = 60 + 8 = 68$); this provides a stepping stone to the first strategy presented in *Teaching point 2* for the addition of two two-digit numbers – the strategy in which both addends are partitioned (e.g. $45 + 23 = 40 + 5 + 20 + 3$). An alternative strategy is then explored in which only one of the addends is partitioned (e.g. $45 + 23 = 45 + 20 + 3$); children should be encouraged to compare the different methods and to reason that the latter (partitioning only one addend) is a more efficient strategy. Teaching the 'partition *both* addends' strategy first provides variation, and the opportunity for children to gain a deeper understanding by comparing the two strategies. The 'partition *one* addend' strategy will be useful when children approach subtraction of one two-digit number from another in segment *1.16 Subtraction: two-digit and two-digit numbers*; there only the subtrahend is partitioned.

When choosing examples/questions it is also important to carefully consider the numbers used, to ensure the focus is appropriate for the given step:

- In each step, follow the guidance on whether the tens boundary should or should not be crossed during addition of the ones.
- All totals should remain within 100.

Commutativity and associativity of addition were introduced in segments *1.7 Addition and subtraction: strategies within 10* and *1.11 Addition and subtraction: bridging 10*; as children work through this and the next segment, they will need to draw on their knowledge of these concepts to develop efficient mental strategies. For example, for the calculation $45 + 23$, after partitioning into tens and ones ($40 + 5 + 20 + 3$), first the law of commutativity is applied to rearrange the addends ($40 + 20 + 5 + 3$), then the law of associativity is applied to group the calculation into two separate addition steps (addition of the tens,

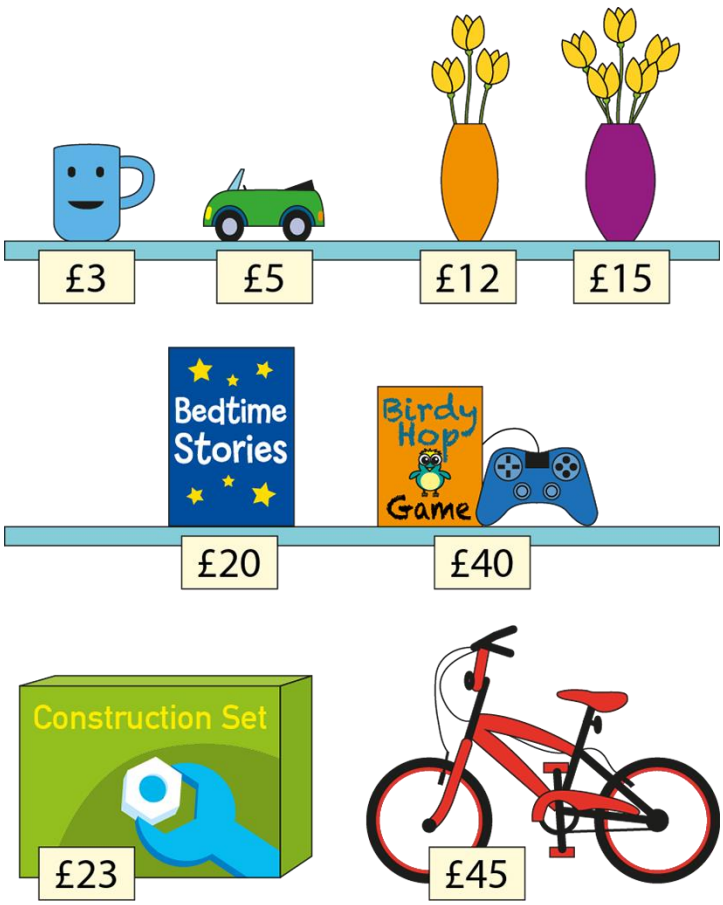
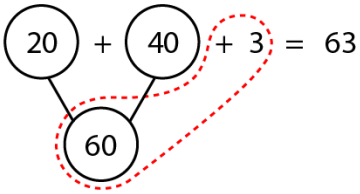
and addition of the ones). Children can also be exposed to the addition of the tens and ones in either order (the tens first or the ones first), or to variation in the order of addends within the tens or within the ones; although this is not the main focus of the segment, it encourages flexibility. This flexibility equips children for various mental strategies.

Throughout, children should be encouraged to use known number facts to add the 'parts'; they should not rely on counting forwards in ones to find the answer. Similarly, the manipulatives should be used as a tool to expose structure; children should not rely on counting the manipulatives to find the answer.

Teaching point 1:

Known strategies can be combined to add two multiples of ten to two single-digit numbers.

Steps in learning

	Guidance	Representations
1:1	Begin this segment by reviewing the following known concepts/strategies: • partitioning a given two-digit number into tens and ones • strategies for bridging ten • adding a single-digit number to a two-digit number • adding two multiples of ten • adding multiples of ten to a two-digit number A useful context is a shop with items priced as single-digit numbers, multiples of ten, teen numbers and other two-digit numbers. For the example items shown opposite, appropriate questions include: • *'How much does a mug and a toy car cost?'* (two single-digit numbers) • *'How much does a book and a computer game cost?'* (two multiples of ten) • *'How much does a book and a toy car cost?'* (multiple of ten and a single-digit number) • *'How much does a book, a mug and a car cost?'* (multiples of ten and two single-digit numbers) • *'How much do three flowers and a toy car cost?'* (two-digit number and a single-digit number) • *'How much does a book, a computer game and a mug cost?'* (two multiples of ten and single digit-number)	Context for practising known concepts/strategies:  *'How much does a book, a computer game and a mug cost?'*  so £20 + £40 + £3 = £63

	In subsequent steps, children will need to be able to apply several of these skills within a single calculation, so pose questions such as those above to allow practise of the different concepts in quick succession. Children should calculate the answers to the questions mentally, and should be able to explain what known fact or mental strategy they used to calculate the answer, for example: • *'How much does a mug and a toy car cost?'* *'I use number facts within ten; three pounds plus five pounds is equal to eight pounds.'* • *'How much does a book and a computer game cost?'* *'I know that two plus four is equal to six, so two tens plus four tens is equal to six tens; the total is sixty.'* As noted in the *Overview of learning*, children should be using known facts/strategies and should not be using counting-on methods. Similarly, you can use manipulatives or visual representations to highlight the mental steps, but they should not be used to aid calculation. It is not appropriate at this stage to get children to select their own items to add, as it is possible to choose items which do not directly support subsequent steps or that require maths they have not yet learnt. Note that, for practice, the tens boundary *could* be bridged when adding single-digit numbers, although this is not included in the example provided.	
1:2	Once it is clear that children have mastered the concepts practised in step *1:1*, and are confident moving between the different types of examples, prepare for the addition of two two-digit numbers, by first working	

with the addition of two multiples of ten and two single-digit numbers. In *Teaching point 2*, these four addends will represent the partitioned parts of the two two-digit numbers; introducing the addition of two two-digit numbers in this way will encourage children to make strong links with prior learning, rather than treating addition of two two-digit numbers as new learning.

You can use the shop scenario to explore these types of question, i.e.: '*How much does a computer game, a book, a toy car and a mug cost?*'

Using Dienes, and abstract equations, draw attention to the strategy of adding the multiples of ten, adding the ones, and then combining those sums to find the overall total:

40 + 20 = 60

5 + 3 = 8

60 + 8 = 68

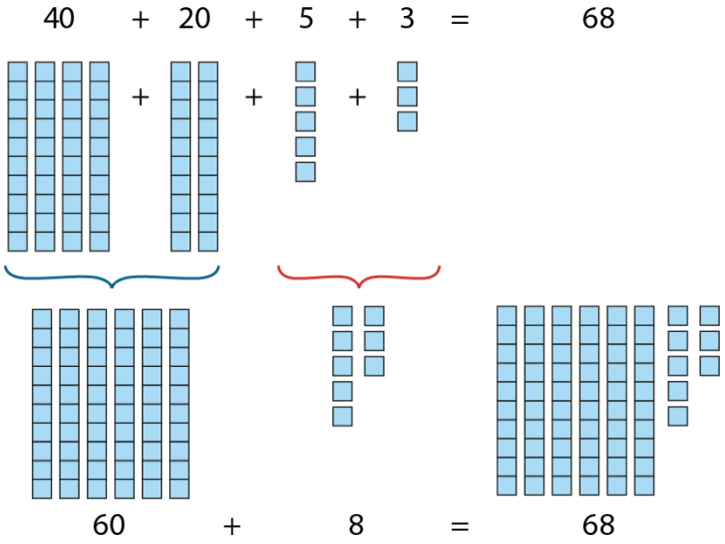
Present the items/calculation in different orders so that children can explore whether the order of the addends affects the total cost. Make sure that you include situations such as:

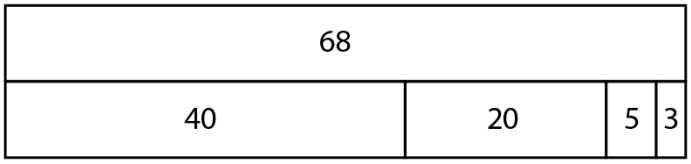
£40 + £5 + £20 + £3

This will facilitate progression to *Teaching point 2*.

Note that the costs of the items in the shop have been chosen carefully to facilitate progression to the next step – finding the total cost of the book (£20), computer game (£40), mug (£3) and toy car (£5) prepares children for finding the total cost of the bike (£45) and the construction set (£23) in the next teaching point. Choose similar combinations to help children make links, avoiding using two single-digit numbers that sum to ten or more.

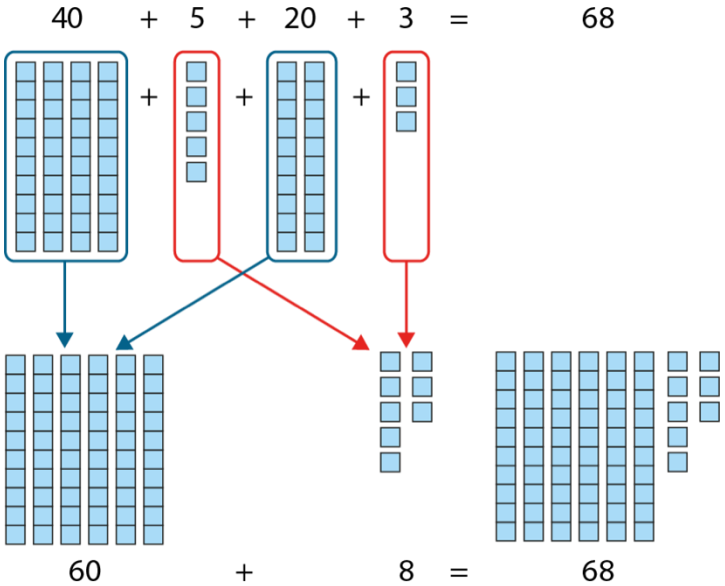
'How much does a computer game, a book, a toy car and a mug cost?'

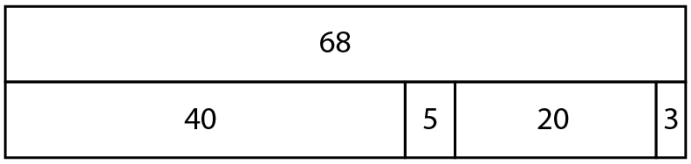




so £40 + £20 + £5 + £3 = £68

'How much does a computer game, a toy car, a book and a mug cost?'

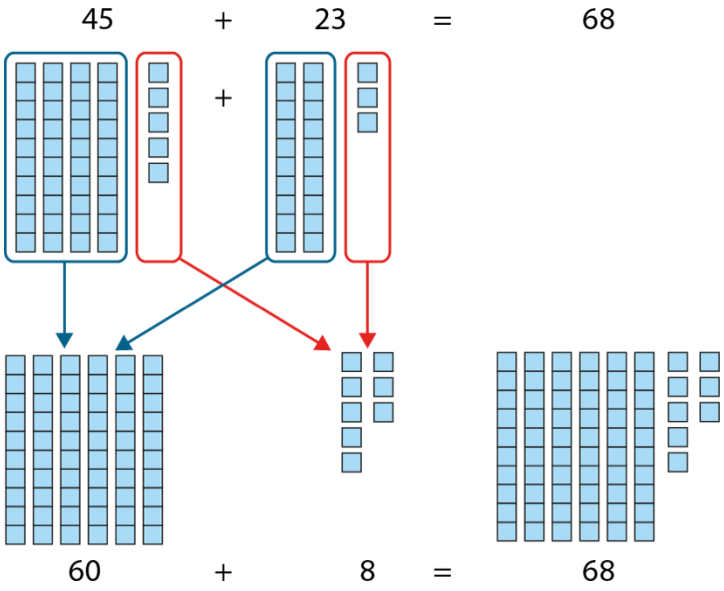


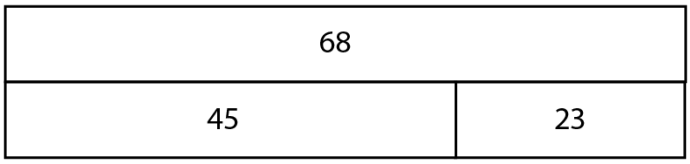
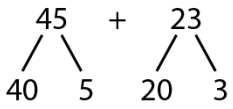
		 so £40 + £5 + £20 + £3 = £68

Teaching point 2:

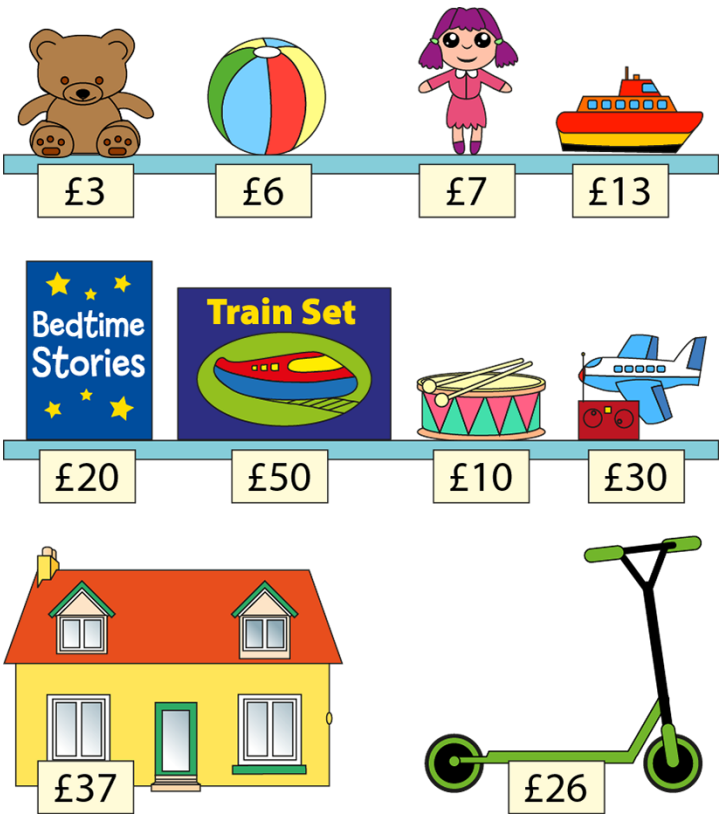
Two two-digit numbers can be added by partitioning one or both of them into tens and ones.

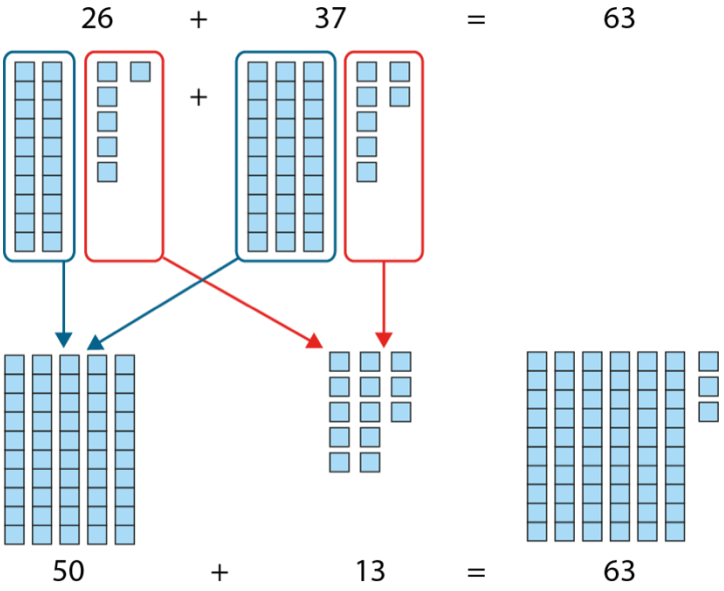
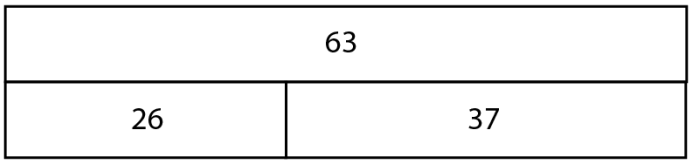
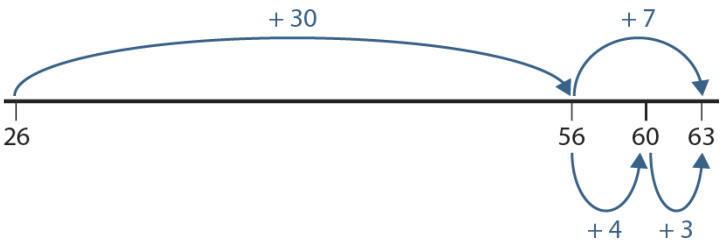
Steps in learning

	Guidance	Representations
2:1	This teaching point is broken down into two stages – problems where a tens boundary is *not* bridged (steps *2:1–2:3*) and problems where the tens boundary *is* bridged (steps *2:4–2:8*). Begin this teaching point by asking children how they could calculate the total cost of the bike (£45) and the construction set (£23). See if children make the link to the example in step *1:2* (*'How much does a book, a computer game, a mug and a toy car cost?'*); if they do not make this link naturally, show them the previous related calculation, and ask: • *'What's the same?'* (We have four tens, five ones, two tens and three ones; the total number of tens is the same and the total number of ones is the same.) • *'What's different?'* (Now we are only adding two numbers.) Highlight that, although addition of two two-digit numbers is new to children, they already know how to complete all steps of the process; this is not really new learning. Once the two-digit numbers have been partitioned into tens and ones the calculation proceeds just as for the previous example. Use Dienes and part–part–whole models, as before, to represent the calculation, using the representations to draw attention to the structure.	*'What is the total cost of the bike and the construction set?'*   Comparing calculations: £40 + £5 + £20 + £3	£45 + £23 • *'What's the same?'* • *'What's different?'* Dienes and part–part–whole model: 

		 so £45 + £23 = £68
2:2	Once children have had the opportunity to work with concrete and pictorial support, move to the use of abstract representations only, showing the children how to jot down partitioning of the addends and express the three addition steps. Encourage children to describe the steps in full sentences as shown opposite.	 *'First I partition the forty-five into forty and five, and the twenty-three into twenty and three.'* 40 + 20 = 60 *'Forty plus twenty is equal to sixty…'* 5 + 3 = 8 *'…five plus three is equal to eight…'* 60 + 8 = 68 *'…and sixty plus eight is equal to sixty-eight.'* £45 + £23 = £68 *'So forty-five pounds plus twenty-three pounds is equal to sixty-eight pounds.'*
2:3	Provide practice in the form of missing number problems, as well as real-life contexts, including different addition structures and measures, for example: • *'There is seventy-five millilitres of water in a glass. Yasmin pours another fifteen millilitres of water into the glass. What is the total volume of water in the glass?'* (augmentation)	Missing number problems: *'Fill in the missing numbers.'* 24 + 63 = ☐ 28 + ☐ = 99 23 + 64 = ☐ ☐ + 71 = 99 ☐ = 52 + 15 64 = 21 + ☐ ☐ = 55 + 12 64 = ☐ + 43

	• *'Mr Garcia has thirty-six footballs. Mr Millet has forty-three tennis balls. How many balls are there altogether?'* (aggregation) Ensure that addition of the ones doesn't bridge the tens boundary, and continue to encourage children to describe the stages of the calculations, using the stem sentences: • ***'First I partition the ___ into ___ and ___, and the ___ into ___ and ___.'*** (partitioning the two-digit addends) • ***'___ plus ___ is equal to ___…'*** (addition of the tens) • ***'…___ plus ___ is equal to ___…'*** (addition of the ones) • ***'…and ___ plus ___ is equal to ___.'*** (addition of the totals of tens and ones) • ***'So ___ plus ___ is equal to ___.'*** (summary of the overall calculation, including units where appropriate) Ensure variation is used within questions where the first and second addends vary in size relative to each other, the position of the equals sign changes, and different measures are used appropriately. Totals should again remain within 100. To assess and promote depth of understanding, present a dòng nǎo jīn problem such as the one shown opposite.	53 + 22 = 52 + ☐ 42 + 34 = 35 + ☐ ☐ + 35 = 45 + 32 ☐ + 17 = 32 + 16 Dòng nǎo jīn: 68 = ☐ + ☐ *'Complete the equation using the following digits.'* 2 4 5 3 *'Can you find another solution?'*
2:4	Once children are secure in their understanding of the addition of two two-digit numbers where addition of the ones does *not* bridge ten, move on to examples where the tens	

	boundary *is* crossed. This can again be related to items in a shop. Begin by reviewing addition of two, three or four numbers, for example: • *'How much does a train set, a drum and a bear cost?'* (£50 + £10 + £3) • *'How much does a train set and a boat cost?'* (£50 + £13) • *'How much does a train set, a ball and a doll cost?'* (£50 + £6 + £7) • *'How much does a book, a ball, an aeroplane and a doll cost?'* (£20 + £6 + £30 + £7) Children already know how to answer all of these questions, but may not have practised moving between questions of the different types. Note that, again, the numbers chosen help children to make links; draw out these links between the different calculations by asking *'What's the same?'* and *'What's different?'* The final question listed above prepares children for the next step – these four addends will represent the partitioned parts of the two two-digit numbers.	Context for practising known concepts/strategies for bridging tens boundaries: 
2:5	Now move on to addition of the two two-digit numbers by asking children how they could calculate the total cost of the scooter (£26) and the doll's house (£37). As in step *2:1*, see if children can make the link to the previous example (*'How much does a book, a ball, an aeroplane and a doll cost?'*); if they do not make this link naturally, show them the previous related calculation, and ask: • *'What's the same?'* (We have two tens, six ones, three tens and seven ones; the total	*'What is the total cost of the scooter and the doll's house?'* 

	number of tens is the same and the total number of ones is the same.) • *'What's different?'* (Now we are only adding two numbers.) Representing the concept with Dienes and part–part–part–whole models, as before, will strengthen the links that children make to previous steps. The only difference in this situation is the combined total of the ones bridges ten. Children already have strategies in place for bridging ten (segment *1.11 Addition and subtraction: bridging 10*).	Comparing calculations: £20 + £6 + £30 + £7 £26 + £37 • *'What's the same?'* • *'What's different?'* Dienes and part–part–whole model:   so £26 + £37 = £63
2:6	The strategy presented in step *2:5* is the same as that used in steps *2:1* and *2:2* (partition *both* the two-digit numbers, add the tens, add the ones, then recombine). Now explore adding the partitioned parts of one of the two-digit numbers to the whole of the other two-digit number, for example: £26 + £30 + £7	26 + 30 + 7 

	This is an alternative strategy, which again builds on previous segments: • The first step involves adding a multiple of ten to a two-digit number, covered in segment *1.14 Addition and subtraction: two-digit numbers and multiples of ten*. • The second step involves adding a single-digit number to a two-digit number. Some children might be tempted to count on in ones to complete this final step of the calculation, but this is an inefficient method and should be avoided. Remind children of the strategies they used for these calculations in *Teaching point 4* of segment *1.13 Addition and subtraction: two-digit and single-digit numbers*.	
2:7	Compare the two strategies (partitioning just *one* of the two-digit numbers versus partitioning *both* of them), asking the questions: • *'What's the same?'* (The overall calculation is the same in each strategy – both give the correct answer; both strategies use prior learning.) • *'What's different?'* (There are fewer steps in the 'partitioning one addend' strategy.) Encourage children to reason that the second strategy (partitioning only one addend) is more efficient and should therefore be used (there are also fewer opportunities for errors).	<table><tr><th>Partitioning both addends</th><th>Partitioning one addend</th></tr><tr><td>26 + 37 20 6 30 7 20 + 30 = 50 6 + 7 = 13 50 + 13 = 63</td><td>26 + 37 30 7 26 + 30 = 56 56 + 7 = 63</td></tr><tr><td colspan="2">so £26 + £37 = £63</td></tr></table>

2:8

Again, provide practice in the form of missing number problems, as well as real-life contexts, including different addition structures, measures contexts, and variation as described earlier, for example:

- *'A sunflower is seventy-five centimetres tall. It then grows another seventeen centimetres. How tall is the sunflower now?'* (augmentation)
- *'Mr Garcia has thirty-eight footballs. Mr Millet has forty-three tennis balls. How many balls are there altogether?'* (aggregation)

Continue to encourage children to describe the steps in each calculation using full sentences.

Include examples for which the ones-digits sum to ten.

To assess and promote depth of understanding, present dòng nǎo jīn problems such as those shown opposite.

Missing number problems:
'Fill in the missing numbers.'

34 + 26 = ☐ 70 = 45 + ☐

36 + 24 = ☐ 70 = ☐ + 25

34 + 27 = ☐ 71 = 45 + ☐ = 92

37 + 24 = ☐ 71 = ☐ + 26

24 + 38 = ☐ 29 + ☐ = 51

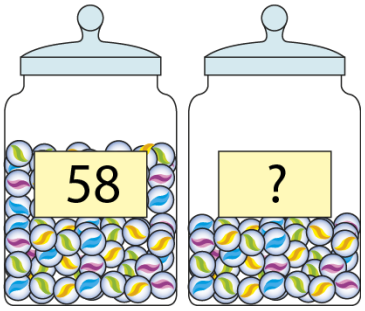
38 + 24 = ☐ ☐ + 22 = 51

Dòng nǎo jīn:

- *'Pippa has eighty-seven marbles. Lotte has fewer marbles than Pippa. In one jar Lotte has fifty-eight marbles. What is the maximum number of marbles Lotte could have in her other jar?'*

Pippa's marbles

Lotte's marbles



- *'Find all the different ways to correctly complete this equation. (Clue: there are more than three.)'*

☐ 2 = 4 ☐ + 4 ☐