

Mastery Professional Development

Number, Addition and Subtraction



1.14 Addition and subtraction: two-digit numbers and multiples of ten

Teacher guide | Year 2

Teaching point 1:

When finding ten more or ten less than any two-digit number, the ones digit does not change.

Teaching point 2:

When ten is added or subtracted to/from a two-digit number, the tens digit changes and the ones digit stays the same.

Teaching point 3:

Knowledge of number facts within ten can be applied to adding or subtracting multiples of ten to/from a two-digit number.

Teaching point 4:

Two-digit numbers can be partitioned in different ways.

Overview of learning

In this segment children will:

- represent and explain addition and subtraction of ten and multiples of ten to/from two-digit numbers
- apply calculation strategies for addition/subtraction within ten to addition or subtraction of multiples of ten to/from two-digit numbers.

This segment builds on the following teaching points in segments *1.8 Composition of numbers: multiples of 10 up to 100* and *1.9 Composition of numbers: 20–100*:

- Known facts for the numbers *within* ten can be used to add and subtract in multiples of ten by unitising. (Segment 1.8, Teaching point 5)
- Each two-digit number can be partitioned into a tens part and a ones part. (Segment 1.9, Teaching point 5)

This segment furthers children's understanding of place value and the composition of two-digit numbers, alongside the development of calculation strategies for adding/subtracting multiples of ten. The segment begins by recapping finding ten more/less than a multiple of ten (from segment 1.8). This is then extended to all ten more/less relationships between 10 and 99, first working with skip-counting and ordinal representations, and then exploring quantity-value representations.

The segment then looks at addition/subtraction of *multiples* of 10 to/from two-digit numbers, applying children's knowledge of unitising and known additive facts within ten (as in segment 1.8) and their understanding of the tens-and-ones structure of two-digit numbers (from segment 1.9). Concrete equipment, such as Dienes, is used to reveal the place-value structure of two-digit numbers and to highlight the change in value of the tens digit on addition/subtraction of multiples of ten. The equipment should be used as a tool to focus attention on the structure and steps of the strategy, rather than as a counting tool for calculating answers.

There is a strong focus, in this segment, on how the tens digits change on the addition/subtraction of multiples of ten to/from two-digit numbers. To draw attention to the emerging patterns, and to allow generalisations to be made more easily, the segment does not include calculations that bridge ten (e.g. $4 + 10$, $14 - 10$, $4 + 30$, $34 - 30$) or calculations that bridge 100 (e.g. $94 + 10$, $104 - 10$, $94 + 30$, $124 - 30$). The former are covered in segments *1.9 Composition of numbers: 20–100* and *1.10 Composition of numbers: 11–19*, while the latter will be covered in segment *1.17 Addition and subtraction: 100 and three-digit numbers*.

Teaching point 1:

When finding ten more or ten less than any two-digit number, the ones digit does not change.

Steps in learning

	Guidance	Representations																																				
1:1	<p>In segment <i>1.8 Composition of numbers: multiples of 10 up to 100</i>, children learnt to count forwards and backwards in multiples of ten, find ten more/less than a given multiple of ten, and add/subtract ten from a given multiple of ten. Begin this teaching point by revisiting counting forwards and backwards in multiples of ten, and finding ten more/less than a multiple of ten, but now (in preparation for the next steps) draw attention to the patterns in the digits.</p> <p>Ask questions of the form:</p> <ul style="list-style-type: none">• <i>‘What is ten more than forty?’</i>• <i>‘What is ten less than fifty?’</i> <p>Then, use a number line, hundred square or Gattegno chart to show the number that is ten more/less.</p> <p>Ensure that the visual representations are used to emphasise the underlying mathematical structure, rather than as a tool for ‘finding the answer’. Draw attention to what is the same and what is different between a given multiple of ten and the number ten more/less. Use the generalised statements:</p> <ul style="list-style-type: none">• <i>‘When we find ten more, the tens digit changes and the ones digit stays the same.’</i>• <i>‘When we find ten less, the tens digit changes and the ones digit stays the same.’</i>	<ul style="list-style-type: none">• <i>‘What is ten more than forty?’</i>• <i>‘What is ten less than fifty?’</i> <table><tr><td>1000</td><td>2000</td><td>3000</td><td>4000</td><td>5000</td><td>6000</td><td>7000</td><td>8000</td><td>9000</td></tr><tr><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td><td>600</td><td>700</td><td>800</td><td>900</td></tr><tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	1000	2000	3000	4000	5000	6000	7000	8000	9000	100	200	300	400	500	600	700	800	900	10	20	30	40	50	60	70	80	90	1	2	3	4	5	6	7	8	9
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1:2	<p>Now work towards extending the generalised statements from step 1:1 to all ten more/less relationships between 10 and 99. Note, as described in the <i>Overview of learning</i>, that moving from,</p>																																					

for example, 4 to 14 (or vice versa) and from, for example, 94 to 104 (or vice versa) are not included here.

Begin with 'skip counting' on and back in tens from *any* two-digit number, using the dual-naming approach described in segment 1.9 *Composition of numbers: 20–100*, for example:

- 'fourteen, twenty-four, thirty-four...' and 'one-ten-four, two-ten-four, three-ten-four...'
- 'seventy-six, sixty-six, fifty-six...' and 'seven-ten-six, six-ten-six, five-ten-six...'

Use chanting linked to the numerals represented on:

- number lines
- hundred squares (ensure that children understand that there is a count of ten between each chanted number – for example, 'fourteen, twenty-four, thirty-four' – by counting in ones before moving to skip counting 'down a column')
- a Gattegno chart (tap the initial single-digit number, then tap 10 and the single-digit number, then tap 20 and the single-digit number, etc.).

Remember that the physical act of tapping on the representation is important for children to do themselves, at times. For now, the focus should be on the pattern in the language and gestures.

Also practise this counting using real-life contexts, such as money, length, mass, capacity, etc. At this point, the practice will still be quite procedural; the purpose is that children become familiar with using the pattern of numbers in a variety of situations which can then be used and built upon at a later stage. When using money as a

Tapping the Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

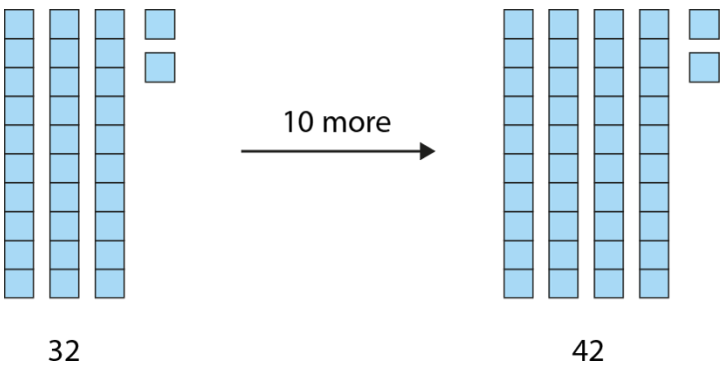
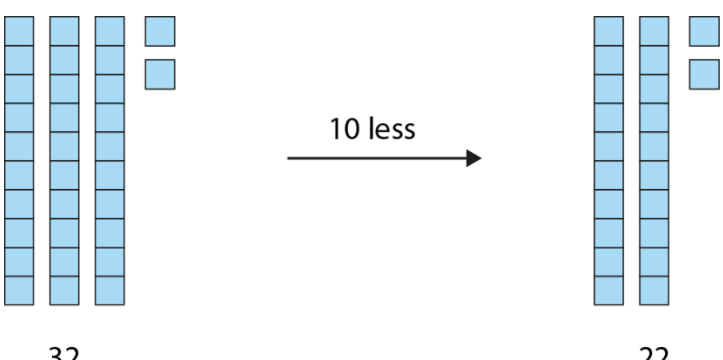
- 'fourteen, twenty-four, thirty-four...'
- 'one-ten-four, two-ten-four, three-ten-four...'

Real-life context:



- 'Forty-two pence, thirty-two pence, twenty-two pence...'
- 'Four-ten-two pence, three-ten-two pence, two-ten-two pence...'

	<p>context, use single pennies for the ones (as shown above), so that the quantity value of the numbers is more evident.</p> <p>During this counting practice, irrespective of which supporting representation is being used, occasionally stop the counting to ask children what number comes next, and why.</p>																																																																																																					
1:3	<p>Keeping to relationships between 10 and 99, and building on the previous step, work on identifying ten more/less than a given number, asking pairs of questions such as:</p> <ul style="list-style-type: none">• ‘What is ten more than forty-two?’• ‘What is ten less than fifty-two?’ <p>Use a number line, hundred square or Gattegno chart to confirm the answer, describing the relationships with the following stem sentences:</p> <ul style="list-style-type: none">• ‘Ten more than ____ is ____. ____ is ten more than ____.’• ‘Ten less than ____ is ____. ____ is ten less than ____.’ <p>Initially work with pairs of related numbers as shown above, then extend practice to just ten more, or just ten less, than a given number.</p> <p>Ensure that the visual representations are used to emphasise the underlying mathematical structure, rather than as a tool for ‘finding the answer’. Draw attention to what is the same and what is different between a given number and the number ten more/less using the generalised statements introduced in step 1:1.</p>	<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr><tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr><tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr><tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr><tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr><tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr></table> <ul style="list-style-type: none">• ‘Ten more than forty-two is fifty-two.’• ‘Ten less than fifty-two is forty-two.’• ‘To find ten more we increase the tens digit by one; the ones digit stays the same.’• ‘To find ten less we decrease the tens digit by one; the ones digit stays the same.’	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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<p>1:4</p>	<p>Most of the representations used so far are based on an ordinal understanding of number. Now examine ten more and ten less using children's understanding of the cardinality (quantity value) of the numbers. Children can manipulate Dienes, adding and removing tens blocks and using stem sentences of the form:</p> <ul style="list-style-type: none"> • 'We had ___ tens and ___ ones. Ten more gives us ___ tens and ___ ones.' • 'We had ___ tens and ___ ones. Ten less gives us ___ tens and ___ ones.' <p>Again, draw attention to the change in the tens digit, and the fact that the ones digit remains the same, continuing to use the generalised statements from step 1:1.</p>	 <p>32 42</p> <p><i>'We had three tens and two ones. Ten more gives us four tens and two ones.'</i></p>  <p>32 22</p> <p><i>'We had three tens and two ones. Ten less gives us two tens and two ones.'</i></p>
<p>1:5</p>	<p>Finally, provide children with practice, as shown below, including:</p> <ul style="list-style-type: none"> • finding both ten less and ten more than a given number • completing sequences on a number line • completing longer missing number sequences. <p>Initially children can use any of the representations from this teaching point for support (number line, hundred square, Gattegno chart or Dienes), but by the end of the teaching point they should be able to identify ten more/less without a scaffold.</p>	
	<p><i>'Fill in the missing numbers.'</i></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>10 less</p> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div> <div style="margin: 0 10px;">←</div> </div> </div> <div style="text-align: center;"> <p>34</p> </div> <div style="text-align: center;"> <p>10 more</p> <div style="display: flex; align-items: center;"> <div style="margin: 0 10px;">→</div> <div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div> </div> </div> </div> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;"> <div style="text-align: center;"> <p>10 less</p> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div> <div style="margin: 0 10px;">←</div> </div> </div> <div style="text-align: center;"> <p>67</p> </div> <div style="text-align: center;"> <p>10 more</p> <div style="display: flex; align-items: center;"> <div style="margin: 0 10px;">→</div> <div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div> </div> </div> </div>	

1.14 Calculation: two-digit +/- tens

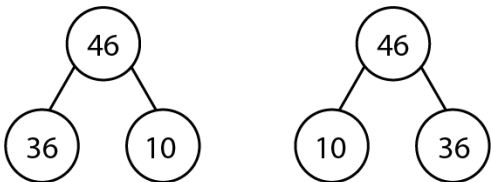
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Teaching point 2:

When ten is added or subtracted to/from a two-digit number, the tens digit changes and the ones digit stays the same.

Steps in learning

	Guidance	Representations																																																																																																																								
2:1	<p>The purpose of this teaching point is to formalise finding ten more/less as addition/subtraction of ten. As in <i>Teaching point 1</i>, keep to relationships between 10 and 99.</p> <p>Begin by introducing equations alongside the concrete/pictorial representations (the number line, hundred square and Dienes are most useful here). To make the link to the skip counting on/back in step 1:2 first mirror a sequence of counting on in tens:</p> <p>$14 + 10 = 24$ $24 + 10 = 34$ $34 + 10 = 44$... $84 + 10 = 94$</p> <p>Then work backwards through the same sequence:</p> <p>$94 - 10 = 84$ $84 - 10 = 74$ $74 - 10 = 64$... $24 - 10 = 14$</p> <p>As in step 1:2, when using a hundred square, ensure that children understand that there is a count of ten between each highlighted number, by counting in ones before moving to addition/subtraction of tens down/up the column.</p> <p>The generalised statements can now be amended to use the language of addition and subtraction:</p>	<table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td></td></tr><tr><td>$14 + 10 = 24$</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>$24 - 10 = 14$</td></tr><tr><td>$24 + 10 = 34$</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>$34 - 10 = 24$</td></tr><tr><td>$34 + 10 = 44$</td><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td><td>$44 - 10 = 34$</td></tr><tr><td>$44 + 10 = 54$</td><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td><td>$54 - 10 = 44$</td></tr><tr><td>$54 + 10 = 64$</td><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td><td>$64 - 10 = 54$</td></tr><tr><td>$64 + 10 = 74$</td><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td><td>$74 - 10 = 64$</td></tr><tr><td>$74 + 10 = 84$</td><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td><td>$84 - 10 = 74$</td></tr><tr><td>$84 + 10 = 94$</td><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td><td>$94 - 10 = 84$</td></tr><tr><td></td><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td><td></td></tr></table>		1	2	3	4	5	6	7	8	9	10		$14 + 10 = 24$	11	12	13	14	15	16	17	18	19	20	$24 - 10 = 14$	$24 + 10 = 34$	21	22	23	24	25	26	27	28	29	30	$34 - 10 = 24$	$34 + 10 = 44$	31	32	33	34	35	36	37	38	39	40	$44 - 10 = 34$	$44 + 10 = 54$	41	42	43	44	45	46	47	48	49	50	$54 - 10 = 44$	$54 + 10 = 64$	51	52	53	54	55	56	57	58	59	60	$64 - 10 = 54$	$64 + 10 = 74$	61	62	63	64	65	66	67	68	69	70	$74 - 10 = 64$	$74 + 10 = 84$	71	72	73	74	75	76	77	78	79	80	$84 - 10 = 74$	$84 + 10 = 94$	81	82	83	84	85	86	87	88	89	90	$94 - 10 = 84$		91	92	93	94	95	96	97	98	99	100	
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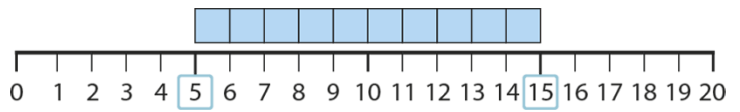
	<ul style="list-style-type: none"> • <i>'When we add ten, the tens digit changes and the ones digit stays the same.'</i> • <i>'When we subtract ten, the tens digit changes and the ones digit stays the same.'</i> <p>Once the link has been made, work through sequences of equations <i>without</i> a visual scaffold, drawing on children's knowledge of composition of number and the generalised statements.</p>	
2:2	<p>When children are comfortable working through sequences, look at the inverse relationship between the addition and subtraction of ten. Find and record ten more than a given number, and then record the inverse for ten less, for example:</p> $26 + 10 = 36 \quad 36 - 10 = 26$ <p>Then present some addition and subtraction calculations out of sequence until children are confident working from any two-digit number, for example:</p> $74 + 10 \quad 53 - 10$	
2:3	<p>Now apply children's understanding of the commutativity of addition (see segment 1.7 <i>Addition and subtraction: strategies within 10</i>), using part–part–whole models for support. The focus here is for children to realise they can use their knowledge of adding ten (ten more) to find, for example, $10 + 36$, rather than thinking of this calculation as finding 36 more than 10.</p> <p>Use stem sentences with 'part' and 'whole' language:</p> <ul style="list-style-type: none"> • <i>'One part is ten, the other part is ____, and the whole is ____.'</i> • <i>'This can be recorded as ten plus ____ is equal to ____, or as ____ plus ten is equal to ____.'</i> 	 <ul style="list-style-type: none"> • <i>'One part is ten, the other part is thirty-six, and the whole is forty-six.'</i> • <i>'This can be recorded as:</i> <ul style="list-style-type: none"> • <i>ten plus thirty-six is equal to forty-six</i> • <i>or as</i> • <i>thirty-six plus ten is equal to forty-six.'</i>

2:4

Now apply subtraction of ten from two-digit numbers to problems with the difference structure. Use a Dienes ten rod aligned with a number line, demonstrating sliding the rod along to find pairs of numbers with a difference of ten. Similarly show pairs of number with a difference of ten on the Gattegno chart.

Then ask children to identify pairs of numbers that have a difference of ten, explaining how they know each pair has a difference of ten.'

Difference structure of subtraction – Dienes and number line:



'I know that five and fifteen have a difference of ten because the tens digit has a difference of one and the ones digit is the same.'

Difference structure of subtraction – Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

'I know that fifty-eight and sixty-eight have a difference of ten because the tens digit has a difference of one and the ones digit is the same.'

2:5

Provide practice in the form of missing number/symbol and missing part/whole problems. Ensure that you vary the position of the equals symbols, as well as the order of the addends.

To promote and assess depth of understanding present đồng nǎo jīn problems such as those shown opposite.

'Fill in the missing numbers.'

$$23 + 10 = \square$$

$$33 - 10 = \square$$

$$33 + \square = 43$$

$$\square - 10 = 33$$

$$\square + 10 = 53$$

$$53 - \square = 43$$

$$10 + 77 = \square$$

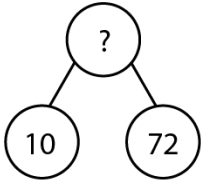
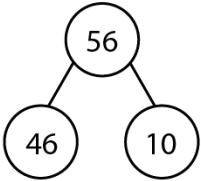
$$87 - 10 = \square$$

$$\square + 67 = 77$$

$$\square - 10 = 67$$

$$10 + \square = 67$$

$$67 - \square = 57$$

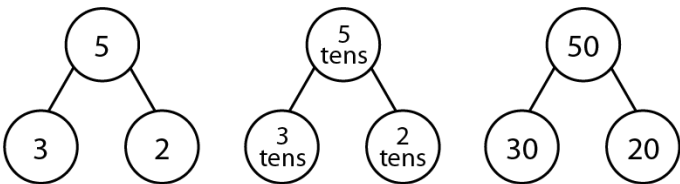
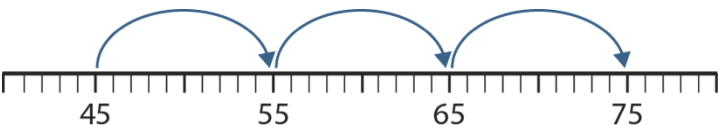
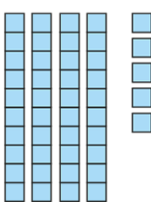
		<p><i>'Fill in the missing numbers or symbols.'</i></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;"> $\square = 10 + 72$ </div> <div style="text-align: center;"> $56 \bigcirc 46 = 10$ </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;"> $72 = \square - 10$ </div> <div style="text-align: center;"> $56 \bigcirc 46 = 10$ </div> </div> <p>Dòng nào jīn:</p> <ul style="list-style-type: none"> 'Jessica writes...' $23 + 10 = 32$ <i>'Without telling her the answer, can you explain how you know she's wrong?'</i> 'Fill in the missing numbers to make this equation correct.' <div style="text-align: center; margin-top: 20px;"> $\square + 10 = \square - 10$ </div> <p><i>'Can you find another way? And another?'</i></p>
<p>2:6</p>	<p>Apply addition/subtraction of ten to/from a two-digit number in a variety of real life contexts, for example:</p> <ul style="list-style-type: none"> 'I have ten blue pens and thirty-six red pens. How many pens do I have altogether?' (aggregation) 'First the bean sprout was twelve centimetres tall, and then it grew ten centimetres. How tall is the bean sprout now?' (augmentation) 'I have eighty-three balls. Ten of them are footballs and the rest are tennis balls. How many tennis balls do I have?' (partitioning) 	

<ul style="list-style-type: none"> • <i>'I had forty-nine pence and then I spent ten pence. How much money do I have left?'</i> (reduction) • <i>'My sunflower is seventy-five centimetres tall and my friend's sunflower is sixty-five centimetres tall. How much taller is my plant?'</i> (difference) • <i>'I scored forty-three points. My friend scored ten more points than me. How many points did my friend score?'</i> (difference) • <i>'My friend has ten pence more than me. He has eighty-two pence. How much money do I have?'</i> (difference) <p>Also, challenge children to generate a variety of stories to match given equations.</p>	
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Teaching point 3:

Knowledge of number facts within ten can be applied to adding or subtracting multiples of ten to/from a two-digit number.

Steps in learning

	Guidance	Representations
3:1	<p>Begin by reviewing addition and subtraction in multiples of ten (segment 1.8 <i>Composition of numbers: multiples of 10 up to 100</i>, Teaching point 5). Emphasise unitising, and using known addition and subtraction facts, revisiting the following stem sentences:</p> <ul style="list-style-type: none"> • 'I know that ___ plus ___ is equal to ___.' • 'So, ___ tens plus ___ tens is equal to ___ tens.' • 'I know that ___ minus ___ is equal to ___.' • 'So, ___ tens minus ___ tens is equal to ___ tens.' 	 <p> $2 + 3 = 5$ $2 \text{ tens} + 3 \text{ tens} = 5 \text{ tens}$ $20 + 30 = 50$ </p> <p> $5 - 3 = 2$ $5 \text{ tens} - 3 \text{ tens} = 2 \text{ tens}$ $50 - 30 = 20$ </p>
3:2	<p>Begin looking at addition of a multiple of ten to a two-digit number (e.g. $45 + 30$) by using a skip-counting approach – counting on in tens from the two-digit number using a number line, hundred square or Gattegno chart (see Teaching point 1).</p>	<p>$45 + 30$</p>  <ul style="list-style-type: none"> • 'Forty-five, fifty-five, sixty-five, seventy-five' • 'Forty-five plus thirty is equal to seventy-five.' <p>$45 + 30 = 75$</p>
3:3	<p>Look at the sum resulting from the skip-counting approach (step 3:2), asking children:</p> <ul style="list-style-type: none"> • 'What do you notice about the tens digits?' • 'What do you notice about the ones digits?' <p>Then demonstrate that a more efficient strategy can be used:</p>	<p>Step 1</p>  <p>'Forty-five is equal to four tens and five ones.'</p> <p>$45 = 40 + 5$</p>

- First, the two-digit number is partitioned into tens and ones.
- Then the tens are added using a known addition fact.
- Finally, the tens and ones are recombined to find the sum.

Use Dienes to represent each step, as shown opposite. Ensure you state the known fact that is being used, and use the equipment to emphasise the structure, rather than counting the tens and ones to 'find the answer'. Write each step, as it is completed, using abstract notation:

$$45 + 30$$

$$45 = 40 + 5$$

$$40 + 30 = 70$$

$$70 + 5 = 75$$

so

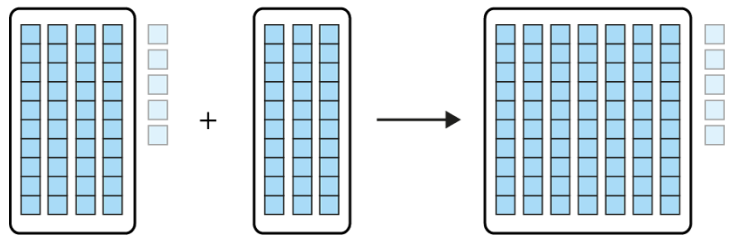
$$45 + 30 = 75$$

If you or the children write the calculation as one continuous equation, ensure that the = symbol is used correctly, with the same total quantity expressed on both sides, for example:

$$45 + 30 = 40 + 30 + 5 = 70 + 5 = 75$$

You can summarise the calculation using a place-value chart or an equation with partitioning as shown opposite.

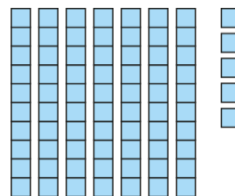
Step 2



- 'I know that four plus three is equal to seven.'
- 'So, four tens plus three tens is equal to seven tens.'

$$40 + 30 = 70$$

Step 3



'Seven tens plus five ones is equal to seventy-five.'

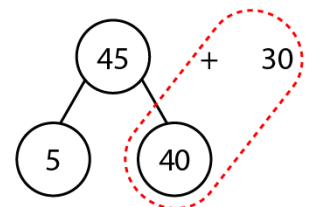
$$70 + 5 = 75$$

Summary

10s	1s
4 + 3	5



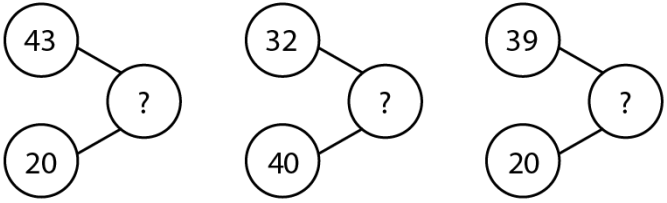
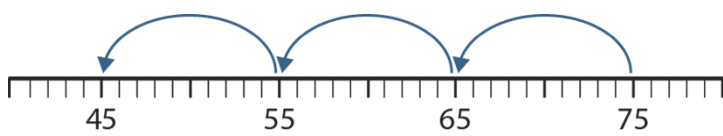
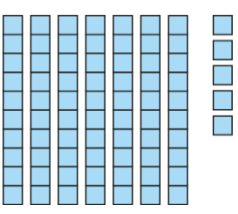
10s	1s
7	5



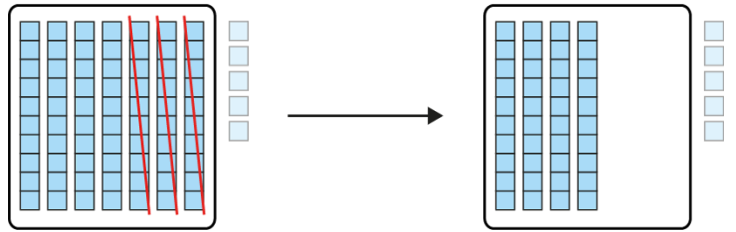
'Four tens and five ones, plus three tens, is equal to seven tens and five ones.'

$$45 + 30 = 75$$

3:4	<p>Work through other calculations in the same way, always making sure the known fact is referred to, varying as follows:</p> <ul style="list-style-type: none">Keep the two-digit number the same and vary the multiple of ten:<ul style="list-style-type: none">$32 + 10$ using $3 + 1$$32 + 20$ using $3 + 2$$32 + 30$ using $3 + 3$etc.Vary the two-digit number and keep the multiple of ten the same:<ul style="list-style-type: none">$15 + 30$ using $1 + 3$$25 + 30$ using $2 + 3$$35 + 30$ using $3 + 3$etc. <p>Work towards use of the following stem sentence to draw attention to the structure:</p> <p>' ___ tens and ___ ones, plus ___ tens, is equal to ___ tens and ___ ones.'</p>	<div><div><div>42</div><div>32</div><div>10</div></div><div><div>52</div><div>32</div><div>20</div></div><div><div>62</div><div>32</div><div>30</div></div></div> <div><div><div>45</div><div>15</div><div>30</div></div><div><div>55</div><div>25</div><div>30</div></div><div><div>65</div><div>35</div><div>30</div></div></div>
3:5	<p>Provide practice in the form of missing number problems as shown opposite, both with sequences of related calculations and 'one-off' calculations. Remember to vary the order of the addends and the position of the equals symbol.</p>	<p><i>'Fill in the missing numbers.'</i></p> <div><div>$14 + 20 = \square$</div><div>$10 + 43 = \square$</div></div> <div><div>$24 + 20 = \square$</div><div>$20 + 43 = \square$</div></div> <div><div>$34 + 20 = \square$</div><div>$30 + 43 = \square$</div></div> <div><div>$\square = 28 + 50$</div><div>$\square = 28 + 60$</div><div>$\square = 28 + 70$</div></div>

		$47 + 20 = \square$ $30 + 34 = \square$ $76 = \square + 20$ $62 = 22 + \square$ 
3:6	<p>Now move on to subtraction of a multiple of ten from a two-digit number, following the same progression as for addition (steps 3:2 to 3:5).</p> <p>As for addition (step 3:2), deliberately start with the less-efficient method of skip counting back in tens, using a number line, hundred square or Gattegno chart.</p>	$75 - 30$  <ul style="list-style-type: none"> 'Seventy-five, sixty-five, fifty-five, forty-five.' 'Seventy-five minus thirty is equal to forty-five.' $75 - 30 = 45$
3:7	<p>Then, demonstrate the more efficient method, using Dienes equipment to represent the steps. Draw attention to the known fact being used to subtract the multiples of ten.</p> <p>Avoid writing out equations such as:</p> $75 - 30 = 70 - 30 + 5 = 40 + 5$ <p>Children will be unfamiliar with addition and subtraction symbols appearing in the same expression ($70 - 30 + 5$), so this could distract from the structure.</p>	<p>Step 1</p>  <p>'Seventy-five is equal to seven tens and five ones.'</p> $75 = 70 + 5$

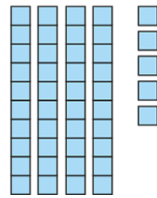
Step 2



- 'I know that seven minus three is equal to four.'
- 'So, seven tens minus three tens is equal to four tens.'

$$70 - 30 = 40$$

Step 3



'Forty tens and five ones is equal to forty-five.'

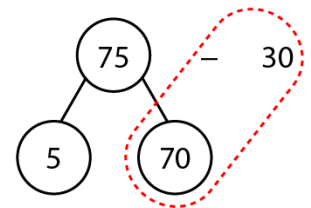
$$40 + 5 = 45$$

Summary

10s	1s
7 - 3	5

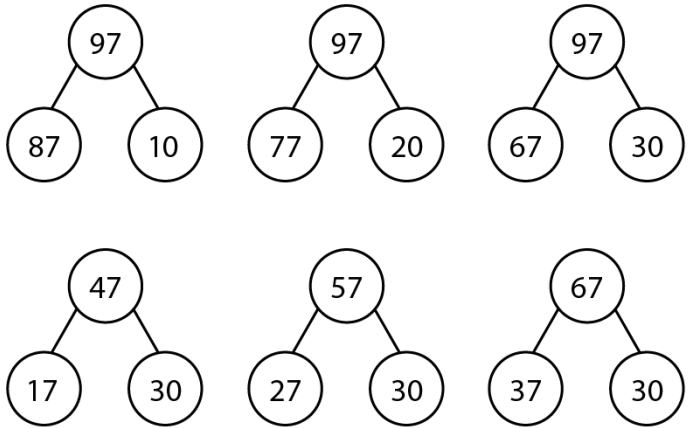
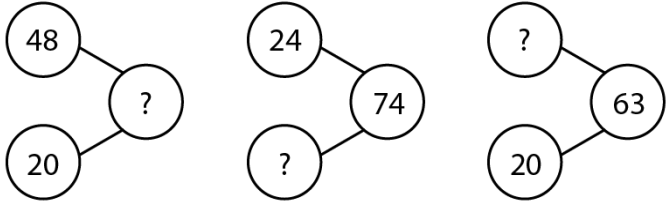


10s	1s
4	5



'Seven tens and five ones, minus three tens, is equal to four tens and five ones.'

$$75 - 30 = 45$$

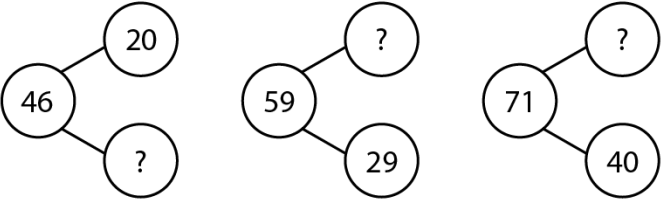
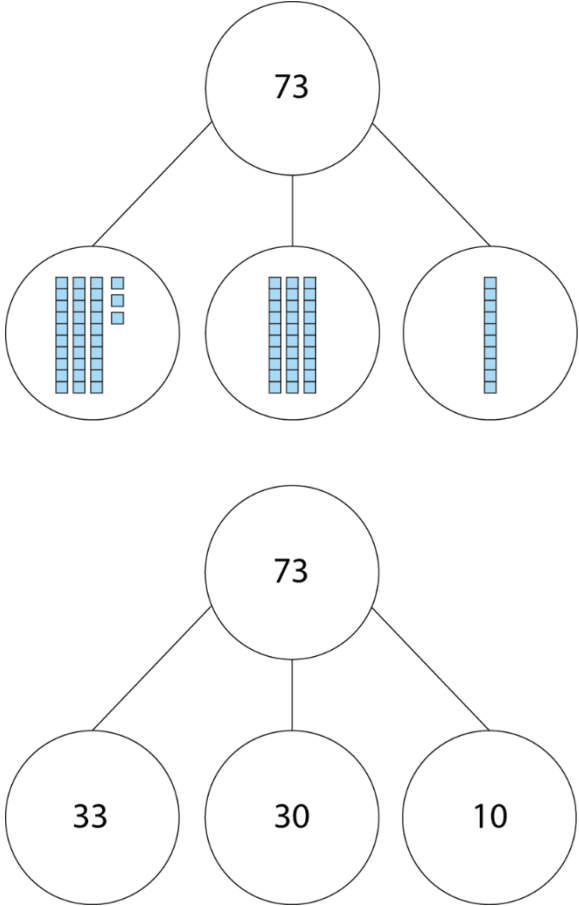
<p>3:8</p>	<p>Work through other calculations in the same way, always making sure the known fact is referred to, varying as follows:</p> <ul style="list-style-type: none"> Keep the two-digit number the same and vary the multiple of ten: <ul style="list-style-type: none"> $97 - 10$ using $9 - 1$ $97 - 20$ using $9 - 2$ $97 - 30$ using $9 - 3$ etc. Vary the two-digit number and keep the multiple of ten the same: <ul style="list-style-type: none"> $47 - 30$ using $4 - 3$ $57 - 30$ using $5 - 3$ $67 - 30$ using $6 - 3$ etc. 	
<p>3:9</p>	<p>Provide practice in the form of missing number problems as shown opposite, both with sequences of related calculations and 'one-off' calculations. Remember to vary the position of the equals symbol.</p>	<div> $34 - 20 = \square$ $83 - 10 = \square$ </div> <div> $44 - 20 = \square$ $83 - 20 = \square$ </div> <div> $54 - 20 = \square$ $83 - 30 = \square$ </div> <div> $\square = 98 - 50$ $47 - 20 = \square$ </div> <div> $\square = 98 - 60$ $81 - 40 = \square$ </div> <div> $\square = 98 - 70$ $25 = \square - 50$ </div> $42 = 82 - 40$ 

Teaching point 4:

Two-digit numbers can be partitioned in different ways.

Steps in learning

	Guidance	Representations
4:1	<p>Although the representations here are similar to those in <i>Teaching point 3</i>, the focus is now on providing children with practice in partitioning two-digit numbers in different ways. This will deepen children's understanding of number, so that they recognise that no matter how a given number is partitioned, the total is conserved. This, in turn, will support later mental strategies, enabling children to build the flexibility to rearrange numbers to facilitate calculation.</p> <p>Begin by representing a two-digit number with Dienes equipment and place this in the 'whole' section on a part-part-whole model. Discuss and demonstrate different ways to partition the Dienes into the 'part' sections of the diagram so that one of the parts is a multiple of ten. Known facts should be used and referred to rather than using the equipment to count.</p> <p>As you 'discover' each way of partitioning, summarise using numerals on a part-part-whole diagram.</p> <p>Discuss how you know whether you have found all possible ways of partitioning, where one part is a multiple of ten. Link this back to systematically partitioning the numbers to ten (segment 1.4 <i>Composition of numbers: 6–10</i>), starting with one part as ten, then looking at the case where one part is twenty, and so on.</p>	

<p>4:2</p>	<p>Provide children with practice including:</p> <ul style="list-style-type: none"> • finding all ways to partition a given two-digit number, where one of the parts is a multiple of ten • completing part-part-whole diagrams with missing 'parts' as shown opposite. 										
<p>4:3</p>	<p>Now extend by exploring how two-digit numbers can be partitioned into three parts, where two of the parts are multiples of ten.</p> <p>As before, present practice including:</p> <ul style="list-style-type: none"> • finding different ways to partition a given two-digit number into three parts, where two of the parts are a multiple of ten • completing part-part-whole diagrams with missing 'parts' or 'wholes' as shown opposite. <p>Use a dòng nǎo jīn problem such as that shown on the next page to assess children's fluency and depth of understanding.</p>	 <p>Dòng nǎo jīn:</p> <p><i>'Fill in the missing numbers, using multiples of ten, so that the row and the column both sum to seventy-five.'</i></p> <table border="1" data-bbox="1023 1641 1230 1845"> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td>15</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>				15					
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4:4	<p>To complete this segment, apply addition/subtraction of multiples of ten to/from two-digit numbers in a variety of real life contexts:</p> <ul style="list-style-type: none"> • <i>'First the sunflower was forty-five centimetres tall, then it grew thirty centimetres more. How tall is the sunflower now?'</i> (augmentation) • <i>'I have thirty blue pens, twenty-six red pens and ten black pens. How many pens do I have altogether?'</i> (aggregation) • <i>'I have eighty-three balls. Fifty are footballs and the rest are tennis balls. How many tennis balls do I have?'</i> (partitioning) • <i>'I had a ribbon that was fifty-six centimetres long and then I cut off twenty centimetres. Then I cut off another ten centimetres. How long is the ribbon now?'</i> (reduction) • <i>'I have twenty-eight pence; my friend has seventy-eight pence. How much more money does my friend have?'</i> (difference) • <i>'My bean plant is twelve centimetres tall. My friend's bean plant is thirty centimetres taller than mine. How much taller is my friend's bean plant?'</i> (comparative addition) • <i>'My friend has seventy-four stickers. She has twenty stickers more than me. How many stickers do I have?'</i> (difference) <p>Also, challenge children to write stories to match given equations.</p> <p>To encourage children to think carefully about the tens and the ones, present a dòng não jīn problem of the form: <i>'There is a pile of ten-pence and one-pence coins. First, I take twenty-five pence. Then, I take two more coins. How much money might I have now?'</i></p>	
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